

$$a) (2x^3 - 1)e^{3x}$$

$$b) \left(\frac{x+e^x}{e^x} \right)^3$$

$$6x^2 e^{3x} + (2x^3 - 1)3e^{3x}$$

$$6x^2 e^{3x} + 6x^3 e^{3x} - 3e^{3x} \cdot 3 \left(\frac{x+e^x}{e^x} \right)^2 \left[\frac{e^x(1+e^x) - (x+e^x)e^x}{e^{2x}} \right]$$

$$3e^{3x} [2x^2 + 2x^3 - 1]$$

$$3 \left(\frac{x+e^x}{e^x} \right)^2 \frac{(1-x)}{e^x}$$

$$c) \ln(\ln x)^2$$

$$2 \ln(\ln x)$$

$$\frac{2}{x \ln x}$$

$$d)$$

$$y = x^{\ln x}$$

$$\ln y = \ln(x^{\ln x})$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \frac{\ln x}{x}$$

$$\frac{dy}{dx} = 2x^{\ln x} \left(\frac{\ln x}{x} \right)$$

$$e) y = \ln 3^x$$

$$= x \ln 3$$

$$y' = \ln 3$$

$$f) y = \ln \frac{x e^x}{\sqrt{1+x}}$$

$$\ln x + \ln e^x - \frac{1}{2} \ln(1+x)$$

$$\ln x + x - \frac{1}{2} \ln(1+x)$$

$$\frac{1}{x} + 1 - \frac{1}{2(1+x)}$$

$$\frac{2 + 2x + 2x + 2x^2 - x}{2x(1+x)}$$

$$\frac{2x^2 + 3x + 2}{2x(1+x)}$$

$$g) \ln \sqrt[3]{\cos x} = \frac{1}{3} \ln \cos x$$

$$y' = \frac{1}{3 \cos x} \cdot -\sin x$$

$$= -\frac{1}{3} \tan x$$

$$h) y = x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y} y' = \ln x + 1$$

$$y' = x^x (\ln x + 1)$$

$$i) y = 10^{x^2}$$

$$\ln y = x^2 \ln 10$$

$$\frac{1}{y} y' = 2x \ln 10$$

$$y' = 10^{x^2} \cdot 2x \ln 10$$

$$j) y = \ln x^{\ln x}$$

$$y = (\ln x)^2$$

$$y' = 2 \frac{\ln x}{x}$$

$$k) y = 3 \sec^{-1} \left(\frac{3}{x} \right)$$

$$= 3 \cos^{-1} \left(\frac{x}{3} \right)$$

$$y' = \frac{1}{\sqrt{1 - \frac{x^2}{9}}} \cdot \frac{1}{3}$$

$$= \frac{1}{\sqrt{9 - x^2}}$$

$$l) \cos(\sin^{-1} x)$$

$$= \sin(\sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$\text{II} \quad y = (x^2 - 15)^5$$

$$\frac{dy}{dx} = 6(x^2 - 15)^5 \cdot 2x = 12x(x^2 - 15)^5$$

$$6(1)^5 \cdot 2(4) = 48 = m$$

$$y' = 1 \quad (4, 1)$$

$$x = 4$$

$$y - 1 = 48(x - 4)$$

$$y = 48x - 192 + 1$$

$$y = 48x - 191$$

$$\text{III} \quad \frac{1}{2} \left[x \sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right]$$

$$\frac{1}{2} \left[\sqrt{x^2 + a^2} + \frac{ax}{\sqrt{x^2 + a^2}} + a^2 \frac{(1 + \frac{2x}{\sqrt{x^2 + a^2}})}{x + \sqrt{x^2 + a^2}} \right]$$

$$\frac{1}{2} \left[\frac{x^2 + a^2 + x}{\sqrt{x^2 + a^2}} + \frac{a^2(\sqrt{x^2 + a^2} + x)}{\sqrt{x^2 + a^2}(x + \sqrt{x^2 + a^2})} \right]$$

$$\frac{1}{2} \left[\frac{2x^2 + 2a^2 + 1}{\sqrt{x^2 + a^2}} \right]$$

$$\frac{x^2 + a^2}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$$

IV

$$\text{let } f(x) = \tan^{-1} x$$

$$g(x) = \tan x$$

$$g' = \sec^2 x$$

$$f'(x) = \frac{1}{g'(f(x))}$$

$$f'(x) = \frac{1}{\sec^2(\tan^{-1} x)} = \cos^2(\tan^{-1} x)$$

$$\text{let } \tan^{-1} x = b \quad x \in \mathbb{R} \quad |b| < \frac{\pi}{2}$$

$$\tan(\tan^{-1} x) = \tan b$$

$$x = \tan b$$

$$x \cos b = \sin b$$

$$x^2 \cos^2 b = 1 - \cos^2 b$$

$$\cos^2 b (1 + x^2) = 1$$

$$\cos^2 b = \frac{1}{1 + x^2}$$

$$\cos^2(\tan^{-1} x) = \frac{1}{1 + x^2}$$

Extra credit is 2 of the "not done" problems from section I

100	1
90	6
80	8
70	8
60	4
59	0

$$\begin{array}{r} 2184 \\ \hline 27 \end{array}$$

80.9

Excellent exam and
great results!

101



60

Advanced Placement Calculus

Name _____

Per _____

Date _____

Copy original problem.

Convince *me* that *you* understand the concept!*No Calculators, please.*

Chapter 5 Exam

IOf the twelve offered problems, you are required to do exactly 10. Given y , find $\frac{dy}{dx}$

Your final answers should be fully factored.

(60 pts tot)

a) $(2x^3 - 1)e^{3x}$

b) $\left(\frac{x+e^x}{e^x}\right)^3$

c) $\ln(\ln x)^2$

d) $x^{\ln x}$

e) $\ln 3^x$

f) $\ln \frac{xe^x}{\sqrt{1+x}}$

g) $\ln \sqrt[3]{\cos x}$

h) x^x

i) 10^{x^2}

j) $\ln x^{\ln x}$

k) $3\sec^{-1}\left(\frac{3}{x}\right)$

l) $\cos(\sin^{-1} x)$

IIFind the equation of the tangent line to the curve: $y = (x^2 - 15)^6$ at $x = 4$. (15 pts)**III**Given: $f(x) = \frac{1}{2} \left[x\sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right]$, show that $f'(x) = \sqrt{x^2 + a^2}$ (10 pts)**IV**Use the result of the chain rule on $f^{-1}(f(x)) = x$, (15 pts tot)

That is use: $(f^{-1})'(f(x)) = \frac{1}{f'(x)}$

to develop the derivative of $f(x) = \tan^{-1} x$.

Include the definitions of whatever variables which are needed in any proofs.

Extra Credit ----- 5 pts -----

Identify the two problems from section I which you didn't do as part of the required 10 and do them now for a total of 5 points.

I $f(x) = -2 + \ln x^2$ 5 PTS
 a) since Domain of $\ln$ is $x > 0$ but x is squared so $x < 0$ is ok.
 $\therefore \forall x \neq 0$

b) $-2 + \ln x^2 = 0$
 $\ln x^2 = 2$
 $\ln x^2 = \ln e^2$
 $\therefore x^2 = e^2$
 $x = \pm e$

c) $f(1) = -2 + \ln 1 = -2$ (1, -2)
 $f'(x) = \frac{1}{x^2} \cdot 2x \Big|_{x=1} = \frac{2}{1} = 2 = m$

$y + 2 = 2(x - 1)$
 $y = 2x - 4$

II c) $f(x) = \begin{cases} -\sin^3 x + \sin^3 |x| & \text{if } x < 0 \\ 2\sin^3 x & \text{if } x \geq 0 \end{cases}$

a) $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 2\sin^3 x & \text{if } x \geq 0 \end{cases}$

b) only place to check is at $x = 0$

$f(0) = 0$ ①
 $\lim_{x \rightarrow 0^-} f(x) = 0$ ②
 $\lim_{x \rightarrow 0^+} f(x) = 2 \cdot 0 = 0$ ③
 $\therefore f$ is cont at $x = 0$

c) $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$\lim_{x \rightarrow 0^-} \frac{0 - 0}{x} = 0 \leftarrow \therefore f'(0) = 0$
 $\lim_{x \rightarrow 0^+} \frac{2\sin^3 x - 0}{x} = 0$

IV $\tan(\sin^{-1}(\frac{2}{3})) + \csc^{-1} 3$

$\tan(\sin^{-1}(\frac{2}{3})) = \frac{\sin \sin^{-1}(\frac{2}{3})}{\cos \sin^{-1}(\frac{2}{3})}$

$\frac{\frac{2}{3}}{\sqrt{1 - \frac{4}{9}}}$

$\frac{\frac{2}{3}}{\sqrt{\frac{5}{9}}} = \frac{2}{\sqrt{5}}$

$\csc^{-1} 3 = \sin^{-1}(\frac{1}{3})$
 $\frac{\sin \sin^{-1}(\frac{1}{3})}{\cos \sin^{-1}(\frac{1}{3})}$

$\frac{\frac{1}{3}}{\sqrt{1 - \frac{1}{9}}}$

$\frac{\frac{1}{3}}{\sqrt{\frac{8}{9}}} = \frac{1}{2\sqrt{2}}$

V $g'(2) = \frac{1}{f'(g(2))}$ where $g = f^{-1}$

a) $g(2) = ?$ ie $f(\text{what})$ is 2; $x = 1$
 so $\frac{1}{f'(1)} = \frac{1}{4}$

b) $f^{-1}(f^{-1}(1))$
 $f^{-1}(2)$

$\tan(\sin^{-1}(\frac{2}{3}) + \csc^{-1} 3) =$

$\frac{\frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{2}}}{1 - \frac{2}{\sqrt{5}} \cdot \frac{1}{2\sqrt{2}}} = \frac{4\sqrt{2} + \sqrt{5}}{2\sqrt{10} - 2}$

$\frac{4\sqrt{2} + \sqrt{5}}{2\sqrt{10} - 2} \approx 1.82514$

$$V a) y = (\tan^{-1} x)^{\cos x}$$

$$\ln y = \cos x \ln(\tan^{-1} x)$$

$$\frac{1}{y} \frac{dy}{dx} = -\sin x \ln(\tan^{-1} x) + \frac{\cos x}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \left| \frac{\sqrt{2}}{2} \left(-\frac{\sqrt{2}}{2} \ln(1) + \frac{\frac{\sqrt{2}}{2}}{1-1} \cdot \frac{1}{1+\frac{\pi^2}{4}} \right) \right|_{x=\frac{\pi}{4}}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{16}{16+\pi^2} = \frac{8\sqrt{2}}{16+\pi^2}$$

$$b) y = x^{x^x} \quad g = x^x \quad \ln g = x \ln x$$

$$\ln y = x^x \ln x$$

$$\frac{1}{g} g' = \ln x + 1$$

$$g' = x^x (\ln x + 1)$$

Note $x^{x^x} \neq (x^x)^x$

so you can't use $x^x \ln x^x$ etc,

$$\frac{1}{y} \frac{dy}{dx} = \ln x (x^x (\ln x + 1)) + x^x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = x^{x^x} \left[\ln x (x^x (\ln x + 1)) + x^{x-1} \right]$$

$$16 [\ln 2 (4 (\ln 2 + 1)) + 2] = 97.2911 \bar{2}$$

because $(x^x)^x = x^{x^2}$

$$c) y = e^x + x^e$$

$$\frac{dy}{dx} = e^x + e x^{e-1}$$

$$= e^{\ln 2} + e (\ln 2)^{e-1}$$

$$= 2 + e (\ln 2)^{e-1}$$

moving this down -2 because the $e-1$ is applied to the $(\ln 2)$ and not to the 2.

Extra: let $x = \text{The number}$

$$\frac{1}{x} = x - 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

so it is possible and 2 numbers satisfy the number is either $\frac{1+\sqrt{5}}{2}$ or $\frac{1-\sqrt{5}}{2}$

10/25/01

100	1
90	2
80	8
70	7
60	2
59	3

101
↓
29

~~107~~
1720 78.2
22

didn't include
Lily chubs 29.

Excellent expm.

emphasis was on neatness
and factoring through.

no changes.

Advanced Placement Calculus

Copy original problem.

Convince *me* that *you* understand the concept!

No Calculators, please.

Name Chloe de S

Per _____

Date 2008

10/25/08

Chapter 5 Exam

I Let f be the function defined by $f(x) = -2 + \ln x^2$.

- For what real numbers is f defined?
- Find the zeros of f .
- Write an equation for the line tangent to the graph of f at $x = 1$.

(total 25 pts)

II Given $f(x) = \sin^3 x + \sin^3 |x|$

- Rewrite f as a piecewise function.
- Prove f is continuous for all x .
- Using the definition of the derivative, find $f'(0)$.

(tot 25 pts)

III Given f is a one-to-one continuous function with the values in the table:

x	$f(x)$	$f'(x)$
1	2	4
2	1	6

(10 pts tot)

- Determine the value of the derivative of $f^{-1}(x)$ evaluated at $x = 2$.
- Determine the value of $f^{-1}(f^{-1}(1))$.

IV

Find $\tan(\sin^{-1}(\frac{2}{3}) + \csc^{-1} 3)$ (Do not rationalize.)

(10 pts)

V

Given y , find $\frac{dy}{dx}$ evaluated at the indicated point.

a) $y = (\tan^{-1} x)^{\cos x}$ Find $\frac{dy}{dx} \Big|_{x = \frac{\pi}{4}}$

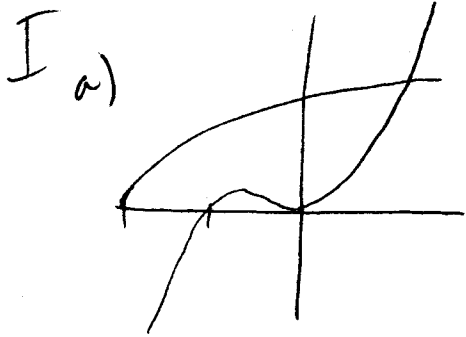
b) $y = x^{x^x}$ Find $\frac{dy}{dx} \Big|_{x=2}$

c) $y = e^x + x^e$ Find $\frac{dy}{dx} \Big|_{x = \ln 2}$

Extra Credit ----- 5 pts -----

"A number has the property that its reciprocal is one less than the number."

Is this possible? If so, how many numbers satisfy the quote. If not, explain how you know that.



b) There appears to be an intersection of the two equations suggesting that there should be a solution.

c) let $f(x) = x^3 + x^2 - \sqrt{x} + 2$

d) let $m=0$

$f(-1) = -1 - 1 - 1 = -3$ still neg.

$f(2) = 8 + 4 - 2 = 10$ positive.

$f(-1) < 0$ and $f(2) > 0$ and m is between them. Thus, since f is a continuous fcn on a closed interval, it follows from the IVT that there exists $\bar{x}$ such that $f(\bar{x}) = 0$.

Thus $\bar{x}^3 + \bar{x}^2 - \sqrt{\bar{x}} + 2 = 0$
 $\bar{x}^3 + \bar{x}^2 = \sqrt{\bar{x}} - 2$

Thus $\bar{x}^3 + \bar{x}^2 - \sqrt{\bar{x}} + 2 = 0$

$\bar{x}^3 + \bar{x}^2 = \sqrt{\bar{x}} - 2$

so we have proven that

II $\frac{u}{\sqrt{1+x}} \sqrt{2-x} - 3 \sin^{-1} \sqrt{\frac{2-x}{3}}$

Generally 5 pts per person

$\frac{\sqrt{2-x}}{2\sqrt{1+x}} + \frac{\sqrt{1+x}(-1)}{2\sqrt{2-x}} - \frac{3}{\sqrt{1-\frac{2-x}{3}}} \cdot \frac{1}{2\sqrt{\frac{2-x}{3}}} \cdot \frac{-1}{3}$

$\frac{2-x - (1+x)}{2\sqrt{1+x}\sqrt{2-x}} + \frac{\frac{3}{\sqrt{3-2+x}} \cdot \frac{1}{2\sqrt{2-x}} \cdot \frac{1}{\sqrt{3}}}{\sqrt{3}}$

$\frac{2-x - 1 - x}{2\sqrt{1+x}\sqrt{2-x}} + \frac{3}{2\sqrt{3-2+x}\sqrt{2-x}} \cdot \frac{1}{2\sqrt{1+x}}$

$-2x + 1$
 $-2(x - 2)$
 $2(2-x)$

$\frac{2(2-x)}{2\sqrt{1+x}\sqrt{2-x}}$
 $\frac{\sqrt{2-x}}{\sqrt{1+x}}$

if $a < 0$

$\frac{-a^2 - 3}{a - 4} = 2a$

$-a^2 - 3 = -2a^2 + 8a$

$a^2 - 8a - 3 = 0$

$a = \frac{8 \pm \sqrt{64 + 12}}{2}$

$\frac{a^2 - 3}{a - 4} \stackrel{\text{set}}{=} 2a$
 $a = 4$

$a^2 - 3 = 2a^2 - 8a$

$a^2 - 8a + 3 = 0$

$a = \frac{8 \pm \sqrt{64 - 12}}{2}$

$a = \frac{8 \pm 2\sqrt{13}}{2}$

$a = 4 \pm \sqrt{13}$

$\frac{-a^2 - 3}{a - 4} = 2a$

$-a^2 - 3 = -2a^2 + 8a$

$a^2 - 8a - 3 = 0$

$a = \frac{8 \pm \sqrt{64 + 12}}{2}$

next possible value of $x = 7$
 f gets 13 ypts
 $(a, f(a))$

1/1
 Generally -10

if $m=8$ at the path is involved. $(a, f(a))$

$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x > 0 \end{cases}$

if $a > 0$

$\frac{8 \pm 2\sqrt{19}}{2}$
 7.606
 3.394

$4 \pm \sqrt{19}$
 7.359

reject $4 + \sqrt{19} > 0$

The 3 values are $4 - \sqrt{19}, 4 \pm \sqrt{13}$

10

Decision - 2 pm phm
no substitution - 3

$$a) y = \ln|x + \sqrt{x^2 - 25}| + \ln|x - \sqrt{x^2 - 25}|$$

min
ln answer
and -5
get
very
ans.

$$y = \ln|x^2 - (x^2 - 25)|$$

$$= \ln 25$$

$$y' = 0 = 0$$

$$x = 5$$

$$b) y = (\tan^{-1} x)^{\cos x} \quad \text{ans } (\tan^{-1} x)^{\cos x - 1}$$

$$\ln y = \cos x \ln \tan^{-1} x$$

$$\frac{1}{y} \frac{dy}{dx} = (\sin x) \ln \tan^{-1} x + \frac{\cos x}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$c) y = x^{x^x} \quad 2^2 = 4^2 = 16$$

$$\ln y = x^x \ln x$$

$$\ln(\ln y) = x \ln x^x + \ln(\ln x)$$

$$= x^x \ln x$$

$$\frac{1}{\ln y} \frac{1}{y} \frac{dy}{dx} = \ln x + 1 + \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \ln y \left[\ln x + 1 + \frac{1}{x \ln x} \right]$$

$$x = 2 = 16 \ln 16 \left[\ln 2 + 1 + \frac{1}{2 \ln 2} \right]$$

$$= 64 \ln 2 \left[\ln 2 + 1 + \frac{1}{2 \ln 2} \right]$$

$$\frac{dy}{dx} = 1^{\frac{\sqrt{2}}{2}} \left(-\frac{\sqrt{2}}{2} \right) \ln \tan^{-1} \frac{\sqrt{2}}{2} + \frac{\frac{\sqrt{2}}{2}}{\tan^{-1} \frac{\sqrt{2}}{2}} + \frac{1}{1 + \frac{\pi^2}{4}}$$

Not zero.

$$\frac{8\sqrt{2}}{16 + \pi^2} \quad \frac{\frac{\sqrt{2}}{2} \cdot \frac{16}{16 + \pi^2}}{2}$$

$$d) y = e^x + x e^{\ln(x^x + x^x)} = e^x + x e^{\ln e^x + \ln e^x} = e^x + x e^{2x}$$

$$\frac{dy}{dx} = e^x + e^x e^{-1} \quad x = \ln 2$$

$$= e^{\ln 2} + e(\ln 2)^{e-1}$$

$$2 + e(\ln 2)^{e-1}$$

Extra

$$\text{Jambh rate} = \frac{1}{7}$$

$$\text{Tanzar's rate} = \frac{1}{8}$$

let T = time together

$$\frac{T}{7} + \frac{T}{8} = 1 \text{ Job}$$

$$\frac{15T}{56} = 1$$

$$T = \frac{56}{15} = 4 \frac{3}{5} = 4 \frac{4}{60}$$

The Job would be complete
at 5:44 pm.

60	
90	0
80	
70	5
60	6
59	7

79



45

$$\frac{1142}{18}$$

63.4

60	0	1
90	0	1
80	4	2
70	4	5
60	5	3
59	4	6

$$\frac{1127}{66.3} \quad \frac{1258}{69.9} \quad \frac{2385}{68.1}$$

103



Students were hoping for
"recycled" exam they can
only do problems they have
seen done. They don't know
the most basic concepts!

for example

III They couldn't figure out
what the PT is question
had coord (a, a^2) or $(a, -a^2)$
most took for $2x (ax - 2x)$
out PT the only x value they
saw, the 4, is for x . Thus
No concept understanding at all!

They don't think, constant area
(like do decimal answers) and
cannot analyze anything!
I think I don't have many B students
and fewer A students.

~~Section IV~~
~~Problems 5~~
~~aren't~~
~~all that~~
~~great.~~

they're OK
very difficult
to keep into
calculation.
two problems with
Section IV
IV A vs allowed use of
sq roots
IV B $\tan^{-1}(1) = \frac{\pi}{4}$
not the other
way around!!

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

No decimal answers.

Name _____

Per _____ Date _____

Chapter 5 Exam

I Determine if $x^3 + x^2 = \sqrt{x+2}$ is solvable. (20 pts)

- a) Using a window with $x \in [-2, 2]$ $y \in [-1, 4]$
graph $y = x^3 + x^2$ and $y = \sqrt{x+2}$. Sketch your picture on your paper.
- b) What conclusion can you make concerning the objective of this problem and your results in part (a)?
- c) Define a function, $f(x)$ which is the difference of the two given equations. Find the narrowest interval $[a, b]$ such that a and b are integers and that $f(a)$ and $f(b)$ are integers which have different signs. Explain, of course.
- d) Using the material above in part, prove the objective of this problem. (ie. Prove a solution exists.)
Give a precise answer. Be very specific. Allow for no misinterpretations.

II Show that the derivative of : $(\sqrt{1+x})(\sqrt{2-x}) - 3\sin^{-1}\sqrt{\frac{2-x}{3}}$ is $\frac{\sqrt{2-x}}{\sqrt{1+x}}$ (20 pts)

III Given: $f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$
Find all three points on the graph of f whose tangent lines pass through $(4, 3)$ (20 pts)

IV Given y . Find $\frac{dy}{dx}$ then evaluate $\frac{dy}{dx}$ at the indicated abscissa value. (10 pts ea)

- a) $y = \ln|x + \sqrt{x^2 - 25}| + \ln|x - \sqrt{x^2 - 25}|$ $x = \sqrt{5}$
- b) $y = (\arctan x)^{\cos x}$ $x = \frac{\pi}{4}$
- c) $y = x^{x^x}$ $x = 2$
- d) $y = e^x + x^e$ $x = \ln 2$

Extra Credit ----- 5 pts -----

Jane can clean the bamboo hut in seven hours and Tarzan can clean the hut in eight hours. Being accurate to the nearest minute, if they both start cleaning at 2 p.m., what time will they finish?

I Know TaShun!

Given: $y = f(x)$; $f(3) = 5$; $f'(3) = 7$; $f''(3) = \frac{1}{2}$

Evaluate each of the following at $x = 3$:

a) $\frac{dy}{dx}$

b) $\frac{dy^2}{dx}$

c) $\frac{d^2y}{dx^2}$

d) $\frac{dy}{dx^2}$

e) $\frac{dy^2}{dx^2}$

Ah, life could be a dream

Product Rule, Quotient Rule, Chain Rule

Exponential Functions

And, if that weren't enough
(and don't you think it ought to be)

Inverse Trig Functions !

..... Sha Boom, Sha Boom

ADVANCED PLACEMENT CALCULUS AB CHAPTER 5 2002

Yes. You do need
those old trig notes.
You do still
love Trig,
don't you?

EXAM Chapters 1 - 4
1 R/C Pg 103 - 108
2 Pg 109 1 - 6, 10
3 Pg 114 - 115
1 - 11, 17 - 19, 26
On pblm 4; *No Decimals*
Ln 2 = A; Ln 3 = B **only!**
Find decimal ans. for prob 11

10/14

10 Pg 146 1 - 4, 8 - 10,
13 - 21 odd; 24, 26
also sketch each of the 6
inverse trig fctns and state
domain & range.
also: simplify and/or solve:

$$\sin^{-1}(\sin u) = u$$

$$\tan(\sin^{-1} u) = ?$$

10/ 21

Continue on

"CE, Third Time"

10/ 28

#4 Pg 118 1, 4 - 7, 11 - 14

5 R/C Pg 143 - 146.

10/15

11 Pg 152 - 153
1 - 21 odd, 22, 28

Der of exponential fctns
12, 13 Explain the formula
at the bottom of pg 147. Use
the 2 examples supplied
below to show your under-
standing of the concept..

Very important!

See HW #12, #13

$$f(x) = \sqrt{x} \text{ for example \# 1}$$

$$f(x) = e^x \text{ for example \# 2}$$

10/29

6 & # 7

Pg 125 - 126 16 - 27, 30, 31

Pg 131 - 132 23, 24, 26, 28

Pg 142 1 - 31 odd

for 25-31 simplify completely

#8 "I Know TaShun"

Do Not Use $(\ln x)(\ln x)$ for $(\ln x)^2$ use: $\ln^2 x$

10/16

14 Pg 159 2 - 28

15 Pg 162 GQ CM 2 - 5

Pg 162

GQ Con 1, 3, 6, 8 - 11

Pg 163 RE 1 - 7

#16 Compare & contrast
thm3 pg 157-158 with
lecture.

10/17

17 Pg 163 RE 8 - 23, 50

10/18

18,

Pg 163 24 - 27, 29, 30, 33

Pg 164 52 - 58, 61, 62,

63, 66, 68

pblm 52 evaluate at $x=5$ not

$$x_0 = 5 \quad 10/ 25$$

19, 20, 21, 22, 23

Pick up "3rd Time"

EXAM
CHAPTER

5

10/31

10/30

$$I_A \quad Y = -\frac{1}{3} \sin^{-1}\left(\frac{3}{x}\right) \quad u = \frac{3}{x}$$

$$\frac{du}{dx} = -\frac{3}{x^2}$$

$$Y = -\frac{1}{3} \sin^{-1} u$$

$$\frac{dY}{du} = -\frac{1}{3} \frac{1}{\sqrt{1-u^2}}$$

$$\frac{dY}{dx} = -\frac{1}{3} \frac{1}{\sqrt{1-\frac{9}{x^2}}} \left(-\frac{3}{x^2}\right)$$

$$\frac{dY}{dx} = \frac{1}{\frac{x^2 \sqrt{x^2-9}}{x}} = \frac{1}{x \sqrt{x^2-9}}$$

$$C) \quad Y = 2^{3x} \cdot 5^{7x}$$

$$2 = e^{\ln 2} \quad 5 = e^{\ln 5}$$

$$2^{3x} = e^{3x \ln 2} \quad 5^{7x} = e^{7x \ln 5}$$

$$Y = e^{3x \ln 2 + 7x \ln 5}$$

$$Y' = e^{3x \ln 2 + 7x \ln 5} (3 \ln 2 + 7 \ln 5)$$

$$= 2^{3x} 5^{7x} (\ln 2^3 + \ln 5^7)$$

$$E) \quad Y = \frac{x}{2} \sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

$$Y' = \frac{1}{2} \sqrt{2-x^2} - \frac{x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{2-x^2-x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2-x^2}} =$$

$$= \frac{2-2x^2+2}{2\sqrt{2-x^2}} = \frac{2-x^2}{\sqrt{2-x^2}} = \sqrt{2-x^2}$$

$$s^2 + c^2 = 1$$

$$t^2 + 1 = \sec^2$$

$$t^2 + 1 = x$$

$$t = \pm \sqrt{x-1}$$

$$\frac{dy}{dx} = \frac{1}{x \sqrt{x^2-1}}$$

$$13) \quad Y = \ln[\ln(1+x^2)]$$

$$\text{let } u = \ln(1+x^2)$$

$$\frac{du}{dx} = \frac{2x}{1+x^2}$$

$$Y = \ln u$$

$$\frac{dY}{du} = \frac{1}{u}; \quad \frac{dY}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{2x}{1+x^2} \cdot \frac{1}{\ln(1+x^2)}$$

$$\frac{2x}{(1+x^2) \ln(1+x^2)}$$

$$D) \quad Y = 3^{\frac{1}{3} \sin 3x}$$

$$3 = e^{\ln 3}$$

$$3^{\frac{1}{3} \sin 3x} = e^{\frac{1}{3} \sin 3x \ln 3}$$

$$Y' = \frac{1}{3} \cdot 3 \cos 3x \ln 3 \cdot e^{\frac{1}{3} \sin 3x \ln 3}$$

$$Y' = (\cos 3x \ln 3) 3^{\frac{1}{3} \sin 3x}$$

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{|x| \sqrt{x^2-1}}$$

$$Y = \sec^{-1} x \quad |x| > 1$$

$$\sec Y = x$$

$$\sec Y \tan Y \frac{dY}{dx} = 1$$

$$\frac{dY}{dx} = \frac{1}{\sec Y \tan Y}$$

$$\frac{dY}{dx} = \frac{1}{x(\pm \sqrt{x^2-1})}$$

Since Y' must be > 0 for $x > 1$ when $x > 1$ we use $+$ so we replace $\pm$ with $|x|$

III

$$1) \lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 3x}) \frac{x + \sqrt{x^2 + 3x}}{x + \sqrt{x^2 + 3x}}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - x^2 - 3x}{x + \sqrt{x^2 + 3x}} = \lim_{x \rightarrow \infty} \frac{-3}{1 + \sqrt{1 + \frac{3}{x}}} = \frac{-3}{2}$$

$$B) \lim_{x \rightarrow 0} \frac{\sin \sqrt{3+x} - \sin \sqrt{3}}{x}$$

$$\text{let } f(a) = \sin \sqrt{a} \quad a=3$$

$$C) \lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1 \quad \frac{\cos \sqrt{a}}{2\sqrt{a}} \bigg|_3 = \frac{\cos \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3} \sin \sqrt{3}}{6}$$

IV $y = 3^x$

$$3 = e^{\ln 3}$$

$$3^x = e^{x \ln 3}$$

$$\frac{dy}{dx} = \ln 3 \cdot e^{x \ln 3} = \ln 3 \cdot 3^x \quad x=2$$

$$9 \ln 3$$

$$\left(\frac{2 \ln 3 - 1}{\ln 3}, 0 \right)$$

$$y - 9 = 9 \ln 3 (x - 2)$$

$$y - 9 = (9 \ln 3)x - 18 \ln 3$$

$$y = (9 \ln 3)x + 9 - 18 \ln 3$$

$$0 = (9 \ln 3)x + 9 - 18 \ln 3$$

$$18 \ln 3 - 9 = (9 \ln 3)x$$

$$2 - \frac{1}{\ln 3} = x$$

Extra

$$\text{let } u = x^x$$

$$x = e^{\ln x}$$

$$= x^x = e^{x \ln x}$$

$$\frac{dy}{dx} = [1 + \ln x] e^{x \ln x} = [1 + \ln x] x^x$$

$$y = x^{x^x}$$

$$y = x^u$$

$$x = e^{\ln x}$$

$$x^u = e^{u \ln x}$$

$$\frac{dy}{dx} = \left[(\ln x)(1 + \ln x)x^x + \frac{u}{x} \right] e^{u \ln x}$$

$$\frac{dy}{dx} = \left[(\ln x)(1 + \ln x)x^x + \frac{x^x}{x} \right] x^{x^x} \cdot [1 + \ln x] x^x$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

$$= \frac{dy}{dx} \bigg|_{x=e}$$

$$(2e^e + \frac{e^e}{e}) 2e^e \cdot e^{e^e} = 2 \left(\frac{2e^e + 1}{e} \right) \cdot e^{2e} e^{e^e} = [1 \cdot (1+1)e^e + \frac{e^e}{e}] e^{e^e} \cdot [1+1] e^e =$$

91

87-2

70.8

86

82

no HW collected -

77

75-2

most did not
really prepare

66

also I was ill

62

Several days -

61

52

No body asked any

50

really significant

40

questions.

this is an
Excellent exam

No changes are
necessary.

(981)

NAME

KEY

CALCULUS CHAPT 5

Mr. Mumaugh

I Find Y' (the 1st derivative) for each of the following. (10 pts ea)

A) $Y = -\frac{1}{3} \sin^{-1}\left(\frac{3}{x}\right)$

B) $Y = \ln[\ln(1+x^2)]$

C) $Y = 2^{3x} \cdot 5^{7x}$

D) $Y = 3^{\frac{1}{3} \sin 3x}$

E) $Y = \frac{x}{2} \sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

II Prove:

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{|x| \sqrt{x^2 - 1}} \quad (10 \text{ pts})$$

III Find the following limits (5 pts each)

(hint: all limits do exist)

A) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 3x})$

B) $\lim_{x \rightarrow 0} \frac{\sin \sqrt{3+x} - \sin \sqrt{3}}{x}$

C) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

IV Determine the x-intercept (if any) of the tangent line to $Y = 3^x$ at (2,9) :
(15 pts)V Show that $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$. Be sure to include all restrictions on any variables. (10 pts)

EXTRA CREDIT _____ 5 pts.

GIVEN: $f(x) = x^{x^x}$. Find $f'(e)$.

I Lopto cu

A)

$$y = 5^{\sin x}$$

$$y = e^{\ln 5^{\sin x}}$$

$$= e^{\sin x \ln 5}$$

$$y' = \cos x \ln 5 \cdot 5^{\sin x}$$

B)

$$y = \frac{\tan x}{2x-1}$$

$$y' = \frac{(2x-1)\sec^2 x - 2\tan x}{(2x-1)^2}$$

C)

$$y = \sec \sqrt[3]{x}$$

$$y' = \frac{\sec \sqrt[3]{x} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{3 x^{\frac{2}{3}}}$$

D)

$$y = (1+x^2)^{-1} \tan^{-1} x$$

$$y' = 2x \tan^{-1} x + \frac{1+x^2}{1+x^2}$$

$$= 2x \tan^{-1} x + 1$$

$$E) y = \ln \left(\frac{x^2}{3x^2+5} \right)$$

$$y = 2 \ln x - \ln(3x^2+5)$$

$$y' = \frac{2}{x} - \frac{6x}{3x^2+5}$$

$$\frac{6x^2+10-6x^2}{x(3x^2+5)} = \frac{10}{x(3x^2+5)}$$

II

$$4) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 4x - 1}{x - \frac{\pi}{2}}$$

$$\text{Given } f(x) = \cos 4x$$

$$\text{Then } \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 4x - \cos 4 \cdot \frac{\pi}{2}}{x - \frac{\pi}{2}}$$

5 pts

$$\therefore f'(x) = -4 \sin 4x \Big|_{\frac{\pi}{2}} = 0$$

$$B) \lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 125}{x-2}$$

$$\text{Given } f(x) = (1+x^2)^3$$

$$\text{Then } \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - (1+2^2)^3}{x-2}$$

$$\therefore f'(x) = 3(1+x^2)^2 (2x)$$

x=2 5pts

$$3(5)^2(-4) = 300$$

III

$$f(3) = 4 \quad g(3) = 3$$

$$f'(3) = -1 \quad g'(3) = 2$$

$$A) (fg)' \quad uv = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$6(3)f'(3) + f(3)g'(3)$$

$$3(-1) + 4(2)$$

$$-3 + 8 = 5 \quad 5 \text{ pts}$$

$$B) \left(\frac{f}{g} \right)' \quad \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{6(3)f'(3) - f(3)g'(3)}{(6(3))^2}$$

$$\frac{3(-1) - 4(2)}{9} = -\frac{11}{9} \quad 5 \text{ pts}$$

$$C) (f(g))' \quad f'(u) \frac{du}{dx}$$

$$f'(g(3)) g'(3)$$

$$f'(3) \cdot 2$$

$$-1 \cdot 2$$

$$-2 \quad 5 \text{ pts}$$

IV

A) $f(x) = x^A$
 $f'(x) = A x^{A-1}$
 5 pts

B) $f(x) = A^x$
 $f(x) = e^{\ln A^x}$
 $= e^{x \ln A}$
 $f'(x) = (\ln A) A^x$ 5 pts

C) $f(x) = A^1$
 $f'(x) = 0$
 A^A is a constant 5 pts

D) $f(x) = x^x$
 $f(x) = e^{x \ln x}$
 $x \ln x$
 $\ln x + 1$ 5 pts
 $f'(x) = (\ln x + 1) x^x$

II WTS $\sec^{-1} x = \cos^{-1}(\frac{1}{x})$ $|x| > 1$
 since $\sec^{-1}$ is in $|x| > 1$

Let $y = \sec^{-1} x$

$\sec y = x$

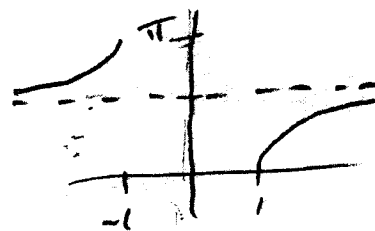
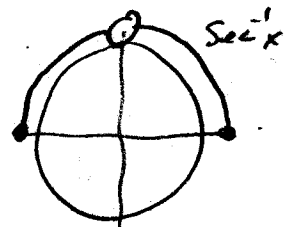
$\frac{1}{\cos y} = x$

$\cos y = \frac{1}{x}$

$y = \cos^{-1}(\frac{1}{x})$

$\sec^{-1} x = \cos^{-1}(\frac{1}{x})$

5 pts.



$0 \leq y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \leq \pi$

Extra.

$f(x) = \ln(\ln(\ln x))$

$f'(e)$

$f(e) = \ln(\ln(\ln e))$
 $= \ln(\ln(1))$
 $= \ln(0)$

$= \text{DNE}$ $\ln$ not defined for 0.
 $\therefore$ NO tangent there is $f'(e)$ DNE

$f'(e^2)$

$f(x) = \ln(\ln(\ln x))$

$f'(x) = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} =$

$\frac{1}{\ln(\ln e^2)} \cdot \frac{1}{\ln e^2} \cdot \frac{1}{e^2}$

$\frac{1}{\ln 2} \cdot \frac{1}{2} \cdot \frac{1}{e^2} = \frac{1}{2e^2 \ln 2}$

10/26/84

103
102-2-4 2475
100 28

88.39

99-2

95
94

92-4

98

90

89

88-3

86

84-2

83

79

77

76

73

70

67

10

8

5

1

Excellent
exam.
especially
at end of
Quarter
and after
the 1st
two hours
exams.

CALCULUS CHAPT 5

Mr. Mumaugh

NAME Calculus Ch 5 1984

Convince me that you do understand the concept !!

I Find Y' (ie the first derivative) for each of the following!

Almost the same as 1986 (10 pts ea)

A) $Y = 5^{\sin x}$

B) $Y = \frac{\tan x}{2x-1}$

C) $Y = \sec \sqrt[3]{x}$

D) $Y = (1+x^2) \tan^{-1} x$

E) $Y = \ln \left\{ \frac{x^2}{3x^2+5} \right\}$

II Using your knowledge of derivatives evaluate : (briefly explain)

(5 pts ea)

A) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 4x - 1}{x - \frac{\pi}{2}}$

B) $\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 125}{x-2}$

III Given: F and G are differentiable fctns, $\forall x \in \mathbb{R}$ and that $F(3) = 4$, $F'(3) = -1$, $G(3) = 3$ and $G'(3) = 2$. Find the numerical value of the following :

(5 pts ea)

(hint: $(F+G)' \Big|_{x=3} = -1 + 2 = 1$. Your method must be even MORE clear than this hint !)

A) $(FG)' \Big|_{x=3}$

B) $\left(\frac{F}{G}\right)' \Big|_{x=3}$

C) $(F[G])' \Big|_{x=3}$

IV Given "A" is a nonzero constant. Find $F'(x)$ for each of the following!

(5 pts ea)

A) $F(x) = x^A$

B) $F(x) = A^x$

C) $F(x) = A^A$

D) $F(x) = x^x$

V Show that $\sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right)$. Be sure to include ALL restrictions on ANY variables.

(5 pts)

EXTRA CREDIT ----- 5 PTS -----

Given : $F(x) = \ln(\ln(\ln(x)))$.

1) Find $F'(e)$

2) Find $F'(e^2)$

I 10pts ea

A) $y = 5^{\cos x}$

$$y = e^{\ln 5^{\cos x}} = e^{\cos x \ln 5}$$

$$y' = 5^{\cos x} (-\sin x) \ln 5$$

$$= -\frac{\sin x}{5^{\cos x}} \ln(5) \sin x$$

C) $y = 2e^{\sqrt{x}}(\sqrt{x}-1)$

$$y' = (\sqrt{x}-1) 2e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + 2e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= e^{\sqrt{x}} - \frac{e^{\sqrt{x}}}{\sqrt{x}} + \frac{e^{\sqrt{x}}}{\sqrt{x}}$$

$$= e^{\sqrt{x}}$$

E) $y = \ln\left(\frac{x^2}{3x^2+5}\right)$

$$= 2\ln x - \ln(3x^2+5)$$

$$y' = \frac{2}{x} - \frac{6x}{3x^2+5}$$

$$\frac{6x^2+10-6x^2}{x(3x^2+5)}$$

$$\frac{10}{x(3x^2+5)}$$

III uv

A) $(fg)' \Big|_{x=3} = Gf' + fG'$

$$(3)(-1) + (4)(2)$$

$$-3 + 8 = 5$$

B) $\left(\frac{f}{g}\right)' \Big|_{x=3} = \frac{Gf' - fg'}{g^2}$

$$\frac{(3)(-1) - 4(2)}{2^2} = \frac{-3-8}{4} = -\frac{11}{4}$$

B) $y = \ln \sqrt{\frac{1+x}{1-x}} = \frac{1}{2} \ln(1+x) - \frac{1}{2} \ln(1-x)$

$$y' = \frac{1}{2(1+x)} - \frac{1(-1)}{2(1-x)}$$

$$\frac{1-x + 1+x}{2(1-x^2)} = \frac{1}{1-x^2}$$

D) $y = (1+x^2) \tan^{-1} x$

$$y' = 2x \tan^{-1} x + \frac{1+x^2}{1+x^2}$$

$$= 2x \tan^{-1} x + 1$$

II A) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 4x - 1}{x - \frac{\pi}{2}}$ This is def of der of $f(x) = \cos 4x$

evaluated at $x = \frac{\pi}{2}$

$$f' = -4 \sin 4x \Big|_{x=\frac{\pi}{2}} = 0$$

B) $\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 125}{x-2}$

This is def of der of $f(x) = (1+x^2)^3$

evaluated at $x=2$

$$f'(2) = 3(1+x^2)^2 \cdot 2x \Big|_{x=2}$$

$$3(5)^2 \cdot 4 = 300$$

C) $(f(G(x)))' \Big|_{x=3} = f'(G(x)) \cdot G'(x)$

chain rule.

$$f'(3) \cdot (2)$$

$$(-1)(2) = -2$$

TO 5pts ea.

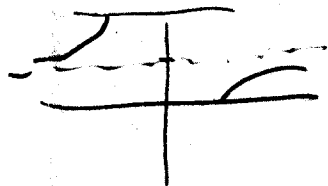
A) $f(x) = x^A$
 $f'(x) = Ax^{A-1}$

B) $f(x) = A^x = e^{\ln A^x}$
 $f'(x) = (\ln A) A^x$

C) $f(x) = A^A$
 $f'(x) = 0$
 (constant ftr)

D) $f(x) = x^x = e^{x \ln x}$

$f'(x) = x^x (\ln x + 1)$



IV w/TT $\sec^{-1}(x) = \cos^{-1}(\frac{1}{x})$

Let $\sec^{-1}(x) = u \quad |x| \geq 1 \quad 0 < u < \pi, u \neq \frac{\pi}{2}$

$x = \sec u$

$\frac{1}{x} = \cos u$

5pts

$\cos^{-1}(\frac{1}{x}) = \cos^{-1}(\cos u)$

$\cos^{-1}(\frac{1}{x}) = u$
 $= \sec^{-1}(x) \quad \text{QED.}$

Extra,

Prove $u'(x) = u(x)$ (like e^x ?)

$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \rightarrow 0} \frac{u(x+h) - u(x)}{h}$

#1 $\lim_{h \rightarrow 0} \frac{u(x)v(h) - u(x)}{h}$

$\lim_{h \rightarrow 0} \frac{u(x)(v(h) - 1)}{h}$

$\lim_{h \rightarrow 0} \frac{u(x) \cdot v(h)}{h} \quad \#2$

$u(x) \lim_{h \rightarrow 0} v(h)$

$u(x) \cdot 1 \quad \#3$

$\therefore$ Since limit is def of $u'(x)$, Der of $u(x)$ is $u(x)$.

1989

96-2

95

92-4 8

91

88-2

83

81

74

73

88%

I need to show ph like
Section II

is.

if $f(x) = (1+x^2)^3$

and if $f'(x)$ were required, using def of der

$$\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - (1+2^2)^3}{x-2}$$

$$\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 5}{x-2}$$

$$\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 125}{x-2}$$

Since this is exactly
the same as $\lim_{x \rightarrow 2} f(x)$
then $f'(2) = 3(1+2^2)^2(2x) = 120$
etc.

10/24/86

100↑	0	98 ↓ 31
90↑	8	
80↑	5	
70↑	4	
60↑	4	
59↓	3	

I graded III, IV very hard. -4 if ~~no~~ NO recognitions of Product rule, etc.

~~was~~

Good exam - 42

$$\frac{1738}{24} = 72.42$$

This is the weakest class of calculus students I've ever had!! This isn't news, I thought they would be weak when I saw them as analysis students last year. Boyle's students have very little serious preparation!

CALCULUS CHAPT 5

Mr. Mumaugh

NAME Ch5 1986

1989

Convince me that you do understand the concept !!

I Find Y' (ie the first derivative) for each of the following! (Simplify final answers)

(10 pts ea)

A) $Y = 5^{\cos x}$

B) $Y = \ln \sqrt{\frac{1+x}{1-x}}$

C) $Y = 2e^{\sqrt{x}}(\sqrt{x} - 1)$

D) $Y = (1+x^2) \tan^{-1} x$

E) $Y = \ln \left\{ \frac{x^2}{3x^2 + 5} \right\}$

II Using your knowledge of derivatives evaluate : (briefly explain)

(5 pts ea)

A) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 4x - 1}{x - \frac{\pi}{2}}$

B) $\lim_{x \rightarrow 2} \frac{(1+x^2)^3 - 125}{x - 2}$

III Given: F and G are differentiable fctns, $\forall x \in \mathbb{R}$ and that $F(3) = 4$, $F'(3) = -1$, $G(3) = 3$ and

$G'(3) = 2$. Find the numerical value of the following :

(5 pts ea)

(hint: $(F+G)' \Big|_{x=3} = -1 + 2 = 1$. Your method must be even MORE clear than this hint !)

A) $(FG)' \Big|_{x=3}$

B) $\left(\frac{F}{G}\right)' \Big|_{x=3}$

C) $(F[G])' \Big|_{x=3}$

IV Given "A" is a nonzero constant. Find $F'(x)$ for each of the following:

(5 pts ea)

A) $F(x) = x^A$

B) $F(x) = A^x$

C) $F(x) = A^A$

D) $F(x) = x^x$

V Prove that $\sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right)$. Be sure to include ALL restrictions on ANY variables. (5 pts)

EXTRA CREDIT ----- 5 PTS -----

Let U and V be functions defined for all real numbers and satisfying the following three conditions.

- 1) $U(X_1 + X_2) = U(X_1) U(X_2)$ (right side is a product.)
- 2) $U(X) - 1 = XV(X)$ (again, right side is a product.)
- 3) $\lim_{x \rightarrow 0} V(X) = 1$.

Prove $U'(X) = U(X)$

I

A) $y = \tan^{-1} \sqrt{1+x^2}$ B) $y = (x+1)^{x+1}$ C) $y = \tan^{-1}(\cos x)$

$y' = \frac{Ax}{x(1+x^2)\sqrt{1+x^2}}$ $y = e^{(x+1)\ln(x+1)}$ $y' = \frac{-\sin x}{1+\cos^2 x}$

$y' = \frac{x}{(2+x^2)\sqrt{1+x^2}}$ $y' = (x+1)^{x+1} \left[\ln(x+1) + \frac{x+1}{x+1} \right]$ $y' = (x+1)^{x+1} [1 + \ln(x+1)]$

D) $y = [\ln(x)]^x$ do like $\ln x^x$ and get $\ln x + 1$

$y = e^{x \ln[\ln(x)]}$ $\ln x + 1 = -2$

$y' = [\ln x]^x \left[\ln(\ln x) + \frac{x}{x \ln x} \right]$

$y' = [\ln x]^x \left[\ln(\ln x) + \frac{1}{\ln x} \right]$

E) $y = \left[\frac{2x}{1-x} + \frac{2}{x-1} \right]^e$ $f) y = \cos^2 e^{3x} + \sin^2 e^{3x}$

$y = \left[\frac{2x-2}{1-x} \right]^e$ $y = 1$

$y = [2]^e$ $\therefore y' = 0$

$y' = 0$

G) $y = \sin^2(\tan^{-1} \sqrt{x})$

$y' = \frac{2 \sin(\tan^{-1} \sqrt{x}) \cos(\tan^{-1} \sqrt{x})}{1+x}$

$= \frac{\sin \tan^{-1} \sqrt{x} \cos(\tan^{-1} \sqrt{x})}{\sqrt{x}(1+x)}$

II $f'(x) = \frac{1}{x^{4/3}}$

$g(x) = f(x^2)$

$g'(x) = \frac{2x f'(x^2)}{x^8+1}$

$\therefore g'(x) = \frac{2x}{x^8+1}$

III

$u(x) = (f \cdot g) \cdot h$

A) $u'(x) = h(gf' + fg') + fgh'$

$= f'gh + fg'h + fgh'$

B) $v(x) = (2x+1)(3x+2)(x+3)^{-2}$

$v'(x) = 2(3x+2)(x+3)^{-2} + 3(2x+1)(x+3)^{-2} - 2(2x+1)(3x+2)(x+3)^{-3}$

15 pts

IV $y = \sin^{-1} x$ $|y| \leq \frac{\pi}{2}$, $|x| \leq 1$ 15 pts

$$\sin y = x$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y \frac{dy}{dx} = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \pm \sqrt{1 - \sin^2 y}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\cos y = \pm \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\cos(\sin^{-1} x)}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

drop $\pm$ since

y is element in 1st or 4th quad, and $\cos$ of any value there is ≥ 0 .

V $y = \frac{e^{Ax}}{A^2 + B^2} (B \cos Bx - A \sin Bx)$

$$y' = \frac{(B \cos Bx - A \sin Bx) A e^{Ax}}{A^2 + B^2} + \frac{e^{Ax}}{A^2 + B^2} [-B^2 \sin Bx - A B \cos Bx]$$

$$= \frac{e^{Ax}}{A^2 + B^2} [A B \cos Bx - A^2 \sin Bx - B^2 \sin Bx - A B \cos Bx]$$

$$= -\frac{e^{Ax}}{A^2 + B^2} (A^2 + B^2) \sin Bx = -e^{Ax} \sin Bx$$
 15 pts

VI $f(1) = 2$ $f'(1) = \frac{1}{3}$

$$[f^{-1}(f(x))]' = \frac{1}{f'(x)}$$

$$[f^{-1}(f(1))]' = \frac{1}{f'(1)}$$

$$f^{-1}(2) = \frac{1}{\frac{1}{3}} = 3$$

$$n = 3 \quad (x, y) = (2, 1) \text{ if } f^{-1}$$

$$y - 1 = 3(x - 2)$$

$$y = 3x - 5$$
 10 pts

extra credit.

$$f\left(\frac{1}{x}\right) = 2x + 1 \Rightarrow f(x) = \frac{2}{x} + 1$$

$$f\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right) + 1 \quad \lim_{x \rightarrow \infty} f(x) = 1$$

$$f(x) = \frac{2}{x} + 1$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{2}{x} + 1 = 1$$

$\therefore$ it does exist and its value is 1.

1987

This is the group who
 "didn't need to be here"
 and thought they could
 survive without
 lecture.
 maybe, (probably not)
 I've got their attention.

90 - 2

74
 73
 72
 71 - 2

Ave
 68.7

65
 62

10/21/87

55
 52
 49

6881 2087

10/29/90

100	0
90	5
80	8
70	9
60	4
59	0

98

}

60

2087
 26

80.6

Copy the original problem.

Convince me that you understand the concept.

Per

Date

CHAPTER # 5

I Find y' (ie. the first derivative) given $y =$: (simplify final answers) (5 pts ea)

A) $\tan^{-1} \sqrt{1+x^2}$

B) $(x+1)^{(x+1)}$

C) $\tan^{-1}(\cos x)$

D) $(\ln x)^x$

E) $\left[\frac{2x}{1-x} + \frac{2}{x-1} \right]^e$

F) $\cos^2(e^{3x}) + \sin^2(e^x)^3$

G) $\sin^2(\tan^{-1} \sqrt{x})$

II Given $f'(x) = \frac{1}{x^4 + 1}$ and $g(x) = f(x^2)$, find $g'(x)$. (10 pts)

III Rules for derivatives. (15 pts)

- A) Derive a "product rule for differentiation" for the product of three functions rather than the usual two functions.

ie. Define $U(x) = F(x)G(x)H(x)$ State $U'(x)$ in a clear and easy to remember form.

- B) Using your "rule", find
- $V'(x)$
- when
- $V(x) = (2x+1)(3x+2)(x+3)^{-2}$

Do not simplify your answer.

IV Given $y = \sin^{-1} x$. Prove $y' = \frac{1}{\sqrt{1-x^2}}$ (15 pts)V Given $y = \frac{e^{Ax}}{A^2 + B^2} (B \cos Bx - A \sin Bx)$. Show that $y' = -e^{Ax} \sin Bx$ (15 pts)VI Given $F(1) = 2$ and $F'(1) = (3)^{-1}$.Determine the equation of the tangent line to $F^{-1}(x)$ at $x = 2$.

Be very sure your method is well documented.

(10 pts)

EXTRA CREDIT = 5 pts =

Given: $F\left(\frac{1}{t}\right) = 2t + 1$. Explain why $\lim_{x \rightarrow \infty} F(x) \Rightarrow DNE$.

I a) $y = (\ln x)^{\ln x} \quad \ln(x^{\ln x}) = -3$ b) $y = \sqrt{\ln x^3} = (3 \ln x)^{\frac{1}{2}}$

$\ln y = \ln^u x \ln(\ln x)$

$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \ln(\ln x) + \ln x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$
 $= \frac{1}{x} (\ln(\ln x) + 1) (\ln x)^{\ln x}$

$y' = \frac{1}{2} (3 \ln x)^{-\frac{1}{2}} \cdot \frac{3}{x}$
 $= \frac{3}{2x \sqrt{\ln x^3}}$ or
 $\frac{3}{2x \sqrt{3} \sqrt{\ln x}} = \frac{\sqrt{3}}{2x \sqrt{\ln x}}$

II $f(1) = 3 \quad g(x) = \sqrt{f(x)}$
 $f'(1) = 4 \quad g' = \frac{1}{2} (f(x))^{-\frac{1}{2}} f'(x)$

$g'(1) = \frac{1}{2} \cdot \frac{1}{\sqrt{3}} \cdot 4^2$
 $= \frac{2\sqrt{3}}{3}$

III $f(x) = \frac{x}{2} \sqrt{4x^2+3} + \frac{3}{4} \ln(2x + \sqrt{4x^2+3})$

$f'(x) = \frac{1}{2} \sqrt{4x^2+3} + \frac{x}{2} \cdot \frac{8x}{2\sqrt{4x^2+3}} + \frac{3}{4} \cdot \frac{1}{2x + \sqrt{4x^2+3}} \cdot 2 + \frac{8x}{2\sqrt{4x^2+3}}$

$= \frac{4x^2+3+4x^2}{2\sqrt{4x^2+3}} + \frac{3}{4} \cdot \frac{1}{2x + \sqrt{4x^2+3}} \cdot \frac{4\sqrt{4x^2+3} + 8x}{2\sqrt{4x^2+3}}$
 $+ \frac{3}{4} \cdot \frac{1}{2x + \sqrt{4x^2+3}} \cdot \frac{1}{2} (\sqrt{4x^2+3} + 2x)$

$\frac{4x^2+3+4x^2+3}{2\sqrt{4x^2+3}}$

IV $f(x) = x^5 + x - 22$

$f(1) = 1 + 1 - 22 = -20$

$f(2) = 32 + 2 - 22 = 12$

$(f^{-1}(12))' = ?$

$g'(f(x)) = \frac{1}{f'(x)}$

$g'(12) = \frac{1}{f'(2)}$

$g'(f(2)) = \frac{1}{f'(2)}$

$= \frac{1}{81}$

$f' = 5x^4 + 1$

$f'(2) = 5 \cdot 16 + 1 = 81$

Try and fail -4
 $\frac{8x^2+6}{2\sqrt{4x^2+3}}$
 $\frac{1}{2} \frac{(4x^2+3)}{\sqrt{4x^2+3}}$
 $\sqrt{4x^2+3}$

IV 4) $\cos(\tan^{-1}x) =$
 let $\tan^{-1}x = u$ $x \in \mathbb{R}$ $14(\frac{\pi}{2})$

$$\tan u = x$$

$$\sin u = x \cos u$$

$$\sin^2 u = x^2 \cos^2 u$$

$$1 - \cos^2 u =$$

$$(x^2 + 1) \cos^2 u = 1$$

$$\cos u = \frac{\pm 1}{\sqrt{1+x^2}}$$

$$\cos u = \frac{1}{\sqrt{1+x^2}} \quad \text{because } u \text{ of } \tan^{-1} x \text{ is } > 0 \text{ there}$$

13) $\tan^{-1}x = \frac{1}{1+x^2}$

$$f(x) = \tan^{-1}x \quad f' = ?$$

$$g(x) = \tan x \quad g'(x) = \sec^2 x$$

$$f' = \frac{1}{g'(f(x))} = \frac{1}{\sec^2(\tan^{-1}x)}$$

$$= \cos^2(\tan^{-1}x)$$

$$= \left(\frac{1}{\sqrt{1+x^2}} \right)^2$$

$$= \frac{1}{1+x^2}$$

"better option" - C

VI a) $f(x) = \sin^{-1}x + \cos^{-1}x$

10 $f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = 0$

b) Since f' is zero, f must be constant.

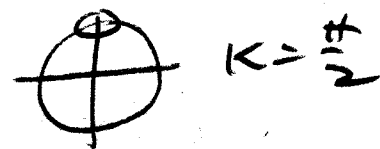
10 c) $\sin(\sin^{-1}x + \cos^{-1}x) = \sin k$

$$\sin(\sin^{-1}x) \cos(\cos^{-1}x) + \sin(\cos^{-1}x) \cos(\sin^{-1}x) = \sin k$$

$$x \cdot x + \sqrt{1-x^2} \cdot \sqrt{1-x^2} = \sin k$$

$$x^2 + 1 - x^2 = \sin k$$

$$1 = \sin k$$



Extra.

$$\ln xy = 3$$

$$e^3 = xy$$

$$\ln \frac{x}{y} = 1$$

$$e = \frac{x}{y}$$

$$ey = x$$

$$e^3 = ey^2$$

$$e^2 = y^2$$

$$y = e \therefore x = e^2$$

Add or replace where
 the fctns. are with
 stuff from trig ch 4

100	0	95
90	3	↓
80	4	
70	6	
60	2	
50	5	47

$$\frac{1437}{20} = 71.85$$

Advanced Placement Calculus

Copy original problem.

Convince *me* that you understand the concept!

Name

CAL Ch5

Per

Date

93

Chapter 5 Exam

I Given $f(x)$, find $f'(x)$. Simplify, if possible.

a) $(\ln x)^{\ln x}$

b) $\sqrt{\ln x^3}$

change to $\sqrt[3]{\ln x^3}$ to improve c) $\sqrt[3]{\ln x}$ 20 pts (10 pts ea)

II Given $f(1) = 3$, $f'(1) = 4$ and $g(x) = \sqrt{f(x)}$, find $g'(1)$.

(10 pts)

III Given $f(x) = \frac{x}{2}\sqrt{4x^2 + 3} + \frac{3}{4}\ln(2x + \sqrt{4x^2 + 3})$, show that $f'(x) = \sqrt{4x^2 + 3}$.

(10 pts)

IV Given $f(x) = x^5 + x - 22$ (Be very sure your method is clear.)

Sum up this problem with the 93 min test (15 pts total) plus how many steps.

a) Find $f(1)$

b) Find $f(2)$

c) Find $(f^{-1})'(12)$

V "Prove-it"s Be *especially* sure that your process is very clear to me! Be sure to include appropriate restrictions on *any* variables.

a) Prove: $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$. Replace with: $\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ (15 pts)

b) Prove: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ (10 pts)

VI Given: $f(x) = \sin^{-1} x + \cos^{-1} x$

(20 pts tot)

a) Show that $f'(x) = 0$.

b) Explain the significance of your answer to part "a".

c) Let $\sin^{-1} x + \cos^{-1} x = k$. Solve for k .

Extra Credit ----- 5 pts -----

Solve the system of equations for the variables x and y :

$$\begin{cases} \ln xy = 3 \\ \ln \frac{x}{y} = 1 \end{cases}$$

$$I \quad y = x^u \sin^{-1} x + \sqrt{1+x^2}$$

$$a) \frac{dy}{dx} = \sin^{-1} x + \underbrace{\frac{x}{\sqrt{1-x^2}} + \frac{-x}{2\sqrt{1-x^2}}}_0$$

$$b) f(x) = 2^{\sin x}$$

$$= e^{\ln 2^{\sin x}}$$

$$f = e^{\sin x \ln 2}$$

$$f' = 2^{\sin x} \cos x \ln 2$$

$$f'(0) = 2^0 \cos 0 \ln 2$$

$$1 \cdot 1 \cdot \ln 2$$

$$E) \text{ no log rule }^{-3}$$

$$f(x) = \ln \left[\frac{\sqrt{2x+1} \sqrt[3]{3x+2}}{(x^2+1)^5} \right]$$

$$f(x) = \frac{1}{2} \ln(2x+1) + \frac{1}{3} \ln(3x+2) - 5 \ln(x^2+1) = -\sin \frac{\pi}{4} \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{2}}{4}$$

$$f'(x) = \frac{x}{2(2x+1)} + \frac{2}{3(3x+2)} - \frac{5 \cdot 2x}{x^2+1}$$

$$f'(0) = \frac{1}{+1} + \frac{1}{2} - 0 = \frac{3}{2}$$

$$c) f(x) = \tan^{-1}(\ln(2x+1))$$

$$f'(x) = \frac{1}{1+(\ln(2x+1))^2} \cdot \frac{1}{2x+1} \cdot 2$$

$$f'(0) = \frac{1}{1+\ln 1} \cdot \frac{1}{1} \cdot 2$$

$$\frac{1}{1} \cdot \frac{1}{1} \cdot 2 = 2$$

$$d) f(x) = \cos(\tan^{-1} x) \quad \text{No sin rule }^{-3}$$

$$f'(x) = -\sin(\tan^{-1} x) \cdot \frac{1}{1+x^2}$$

$$f'(1) = -\sin(\tan^{-1} 1) \cdot \frac{1}{1+1}$$

$$= -\sin \frac{\pi}{4} \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{2}}{4}$$

$$f) f(x) = x^x = e^{\ln x^x}$$

$$f(x) = e^{x \ln x}$$

$$f'(x) = x^x [\ln x + 1]$$

$$f'(e) = e^e [\ln e + 1]$$

$$= e^e [2]$$

$$2e^e$$

$$IIA) \lim_{x \rightarrow 0} \frac{\sin 2x^2}{x^2 + x^3}$$

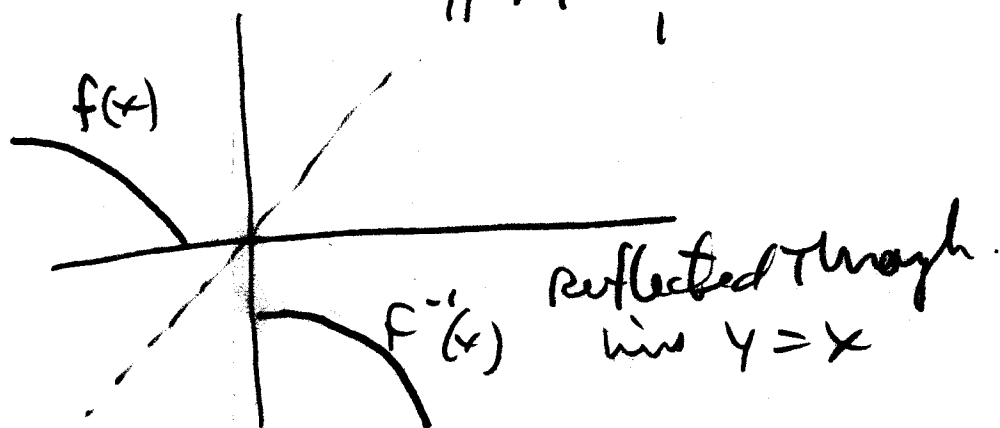
$$\lim_{x \rightarrow 0} \frac{2 \sin 2x^2}{2x^2(1+x)}$$

$$2 \cdot 1 \cdot \frac{1}{1+0} = 2$$

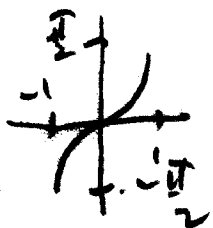
II b) $\lim_{x \rightarrow 0} (1-x)^{\frac{2}{x}}$
 $\lim_{x \rightarrow 0} \left[(1+(-x))^{\frac{1}{-x}} \right]^{-2}$
 e^{-2}
 $\frac{1}{e^2}$

c) $\lim_{x \rightarrow \infty} x \tan \frac{\pi}{x}$
 $\lim_{x \rightarrow \infty} \frac{\pi \frac{\sin \frac{\pi}{x}}{\frac{\pi}{x}}}{\frac{\pi}{x}} \cdot \frac{1}{\cos \frac{\pi}{x}}$
 $\pi \cdot 1 \cdot \frac{1}{1} = \pi$

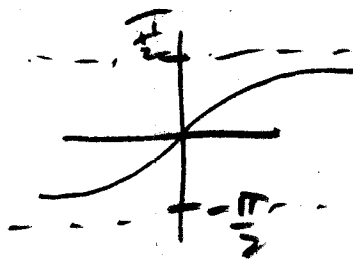
III $f(x) = \sqrt{-1-x}$
 $D_f x \leq -1$
 $R_f f(x) \geq 0$



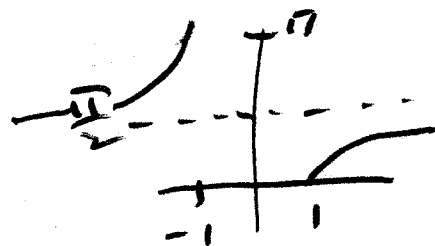
IV a) $f(x) = \sin^{-1} x$
 $D_f |x| \leq 1$
 $R_f |f(x)| \leq \frac{\pi}{2}$
 $f'(x) = \frac{1}{\sqrt{1-x^2}}$



b) $f(x) = \tan^{-1} x$
 $D_f x \in \mathbb{R}$
 $R_f |f(x)| < \frac{\pi}{2}$
 $f'(x) = \frac{1}{1+x^2}$



c) $f(x) = \sec^{-1} x$
 $D_f |x| \geq 1$
 $R_f 0 \leq f(x) \leq \pi, f(x) \neq \frac{\pi}{2}$
 $f'(x) = \frac{1}{|x|\sqrt{x^2-1}}$



Extra

$(\ln x)^2 = \ln x^2$
 $(\ln x)^2 - 2 \ln x = 0$
 $\ln x (\ln x - 2) = 0$

$\ln x = 0$ or $\ln x - 2 = 0$
 $x = 1$ $\ln x = 2$
 $x = e^2$
 $x \in \{1, e^2\}$

92

100	0
90	7
80	3
70	3
60	3
59	6

98
↓
40

$$\frac{1582}{2} = 791$$

Excellent exam.
1 error as marked
on opposite side.

we see that when students
are ~~first~~ faced with new
material they fold.

94

100	0
90	8
80	5
70	1
60	1
59	1

97
↓

$$\frac{1357}{16} = 84.81$$

Still a
great exam!
No changes

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____

Date _____

Convince *me* that *you* understand the concept!

Chapter 5 Exam

94 92

I Perform the indicated operations

(10 pts ea)

- a) Given $y = x \sin^{-1} x + \sqrt{1 - x^2}$. Find $\frac{dy}{dx}$.
- b) Given $f(x) = 2^{\sin x}$. Find $f'(0)$.
- c) Given $f(x) = \tan^{-1}(\ln(2x + 1))$. Find $f'(0)$.
- d) Given $f(x) = \cos(\tan^{-1} x)$. Find $f'(1)$.
- e) Given $f(x) = \ln \left[\frac{\sqrt{2x+1} \cdot \sqrt[3]{3x+2}}{(x^2+1)^5} \right]$. Find $f'(0)$.
- f) Given $f(x) = x^x$. Find $f'(e)$.

II Find the following:

(5 pts ea)

- a) $\lim_{x \rightarrow 0} \frac{\sin 2x^2}{x^2 + x^3}$
- b) $\lim_{x \rightarrow 0} (1 - x)^{\frac{2}{x}}$
- c) $\lim_{x \rightarrow \infty} x \tan \frac{\pi}{x}$

III Given $f(x) = \sqrt{-1-x}$

(Tot 10 pts)

- a) State the domain *and* range of $f(x)$.
- b) Sketch the graph of both $f(x)$ and $f^{-1}(x)$ on the same set of axes.

IV For each of the following state Domain, Range *and* the derivative.

Sketch $f(x)$ on a properly labeled axis.

(5 pts ea)

- a) $f(x) = \sin^{-1} x$
- b) $f(x) = \tan^{-1} x$
- c) $f(x) = \sec^{-1} x$

Extra Credit ----- 5 pts -----

Solve for x :

$$(\ln x)^2 = \ln x^2$$

$$I) y = \tan^{-1}(\sqrt{1+x^2})$$

$$y' = \frac{1 \cdot x}{1 + (1+x^2)} \cdot \frac{1}{\sqrt{1+x^2}}$$

$$\left. \frac{x}{(2+x^2)(\sqrt{1+x^2})} \right|_{x=1} = \frac{1}{6\sqrt{2}} = \frac{\sqrt{2}}{6}$$

$$c) y = (\ln x)^x$$

$$\ln y = x \ln(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(\ln x) + \frac{x}{\ln x} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = (\ln x)^x \left[\ln(\ln x) + \frac{1}{\ln x} \right] \Big|_{x=e^2} = (2)^{e^2} \left[\ln 2 + \frac{1}{2} \right]$$

$$E) y = \ln \left[\frac{\sqrt{2x+1} \sqrt{3x+2}}{(x^2+1)^5} \right]$$

$$= \frac{1}{2} \ln(2x+1) + \frac{1}{3} \ln(3x+2) - 5 \ln(x^2+1)$$

$$y' = \frac{1}{2} \frac{2}{2x+1} + \frac{1}{3} \frac{3}{3x+2} - \frac{5 \cdot 2x}{x^2+1} \Big|_{x=1}$$

$$\frac{1}{3} + \frac{1}{9} - 5$$

$$\frac{5+3-75}{15}$$

$$-\frac{67}{15}$$

$$B) y = (x+1)^{x+1}$$

$$\ln y = (x+1) \ln(x+1)$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(x+1) + \frac{(x+1)}{(x+1)}$$

$$\frac{dy}{dx} = (x+1)^{x+1} [\ln(x+1) + 1] \Big|_{x=1} = 2^2 [\ln 2 + 1]$$

$$D) y = \left(\frac{2x}{1-x} + \frac{2}{x-1} \right)^e$$

$$= \left(\frac{2x-2}{-1-x} \right)^e$$

$$= \left(\frac{2(x-1)}{1-x} \right)^e$$

$$= (-2)^e$$

$$y' = 0 \Big|_{x=2} = 0$$

NO
STYLE
-6

II a) $\sin(\cos^{-1}x) = \sqrt{1-x^2}$
 let $u = \cos^{-1}x$ $|x| \leq 1$
 $0 \leq u \leq \pi$

$\cos u = x$
 $\cos^2 u = x^2$
 $1 - \sin^2 u = x^2$
 $\sin^2 u = 1 - x^2$
 $\sin u = \pm \sqrt{1-x^2}$

$\sin(\cos^{-1}u) = \sqrt{1-x^2}$
 Since u is an element in the 1st or 2nd quad, $\sin > 0$

b) $f(x) = \cos^{-1}x$
 $g(x) = \cos x$ $g' = -\sin x$

$f'(x) = \frac{1}{g'(f(x))}$
 $= \frac{1}{-\sin(\cos^{-1}x)}$

$(\cos^{-1}x)' = \frac{-1}{\sqrt{1-x^2}}$
 using part a.

4) $f(x) = \sin(2\cos^{-1}x)$

① $2\sin \cos^{-1}x \cos(\cos^{-1}x)$
 $(2x\sqrt{1-x^2})'$

$2\sqrt{1-x^2} + \frac{2x(-2x)}{2\sqrt{1-x^2}}$
 $\frac{2-2x^2+2x^2}{\sqrt{1-x^2}}$
 $\frac{2-4x^2}{\sqrt{1-x^2}}$
 $\frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$

$\cos^2(2\cos^{-1}x) \left(\frac{-2}{\sqrt{1-x^2}}\right)$ from part b

$[\cos^2(\cos^{-1}x) - \sin^2(\cos^{-1}x)] \left[\frac{-2}{\sqrt{1-x^2}}\right]$
 $[x^2 - (1-x^2)] \left[\frac{-2}{\sqrt{1-x^2}}\right]$
 $[2x^2 - 1] \left[\frac{-2}{\sqrt{1-x^2}}\right]$ No differences
 $\frac{2-4x^2}{\sqrt{1-x^2}}$ ② 1.155

III ~~$4 \ln x = 3$
 $\ln x \ln 4 = \ln 3$
 $\ln x = \frac{\ln 3}{\ln 4}$
 $x = e^{\frac{\ln 3}{\ln 4}}$
 $= [e^{\frac{\ln 3}{\ln 4}}]^{\ln 4}$
 $x = 3^{\frac{1}{\ln 4}}$~~

$g(p(-x))$
 $g(-p(x))$
 $g(p(x))$
 even

$p(g(-x))$ $p(-x)(g(-x))$
 $p(g(x))$ $-p(x)g(x)$
 even odd
 only III
 "C"

98 NO

100	0
90	0
80	6
70	5
60	1
59	3

89
↓
45

1107
15 73.8

Excellent exam
No changes.

95

100	1
90	1
80	5
70	8
60	2
59	3

102
↓
51

75.05

Exam seems to be just
about long.
Drop part III
and add ~~part 3 to~~
~~as~~ as changed

make III the ~~ext~~
and drop The ~~ext~~

Advanced Placement Calculus

Copy original problem.

Convince *me* that you understand the concept!

Name

Ch ch 5 Exam

Per

Date

9/5/98

Chapter 5 Exam

I

Evaluate the following. No decimals.

(12 pts ea)

a) $y = \tan^{-1} \sqrt{1+x^2}$
Find $y'(1)$

b) $y = (x+1)^{(x+1)}$
Find $y'(1)$

c) $y = (\ln x)^x$
Find $y'(e^2)$

d) $y = \left(\frac{2x}{1-x} + \frac{2}{x-1} \right)^e$
Find $y'(2)$

e) $y = \ln \left[\frac{\sqrt{2x+1} \sqrt[3]{3x+2}}{(x^2+1)^5} \right]$
Find $y'(1)$

II

Derivatives of the inverse trig functions.

(tot 40 pts)

a) Prove: $\sin(\cos^{-1} x) = \sqrt{1-x^2}$

b) Prove: $\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$

c) Compute $\text{nDeriv}(\sin(2 \cos^{-1} x), x, .5)$ where "nDeriv(" is under the "math" menu.

d) Given: $f(x) = \sin(2 \cos^{-1} x)$

- 1) First simplify the function using the "sine add" formula then find the derivative of the result. Evaluate your derivative at $x = .5$.
- 2) Using the chain rule, find the derivative of $f(x)$ as it stands. Evaluate your derivative at $x = .5$.

Extra Credit ----- 5 pts -----

Given: $p(x)$ is a non-zero odd function; $q(x)$ is a non-zero even function

Which of these must be odd: I. $q(p(x))$ II. $p(q(x))$ III. $p(x)q(x)$.

- A) I B) II C) III D) I and II E) none of these

I

A) $y = -\frac{1}{3} \sin^{-1}\left(\frac{3}{x}\right)$

$u = \frac{3}{x} \quad \frac{du}{dx} = -\frac{3}{x^2}$

$y = -\frac{1}{3} \sin^{-1} u$

$\frac{dy}{dx} = -\frac{1}{3} \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = -\frac{1}{3} \frac{1}{\sqrt{1-\left(\frac{9}{x^2}\right)}} \left(-\frac{3}{x^2}\right)$

$= \frac{1}{x^2 \sqrt{x^2-9}}$

$= \frac{1}{x \sqrt{x^2-9}} \quad 10 \text{ pts}$

C) $y = 2^{3x} 5^{-7x}$

$u \quad 2^{3x} = e^{3x \ln 2} \quad \text{or} \quad 5^{7x} = e^{7x \ln 5}$
 $du \quad (3 \ln 2) 2^{3x} \quad \text{or} \quad (7 \ln 5) 5^{7x}$

$5^{7x} (3 \ln 2) (2^{3x}) + 2^{3x} 5^{7x} (-7 \ln 5)$

$2^{3x} 5^{7x} [\ln 8 + \ln 5]$
 10 pts

II

$y = 3^x = e^{x \ln 3}$

$y' = \ln 3 \cdot 3^x \big|_{x=2} = 9 \ln 3$

$y|_{x=2} = 9 \quad m = 9 \ln 3 \quad (2, 9)$

$y - 9 = 9 \ln 3 (x - 2)$

$y = 9 \ln 3 (x - 2) + 9 \quad \text{set}$

$\ln 3 (x - 2) = -1$

$x - 2 = -\frac{1}{\ln 3}$

$x = 2 - \frac{1}{\ln 3} \quad 15 \text{ pts}$

x intercept $\left(2 - \frac{1}{\ln 3}, 0\right)$

B) $y = \ln[\ln(1+x^2)]$

$u = \ln(1+x^2)$

$\frac{du}{dx} = \frac{2x}{1+x^2}$

$\frac{dy}{dx} = \frac{1}{\ln(1+x^2)} \cdot \frac{du}{dx}$

$\frac{2x}{(1+x^2) \ln(1+x^2)} \quad 10 \text{ pts}$

D) $y = 3^{\frac{1}{3} \sin 3x} = e^{\frac{1}{3} \sin 3x \ln 3}$
 $y' = \frac{1}{3} (\cos 3x) (3) 3^{\frac{1}{3} \sin 3x \ln 3}$
 $= (\cos 3x \ln 3) 3^{\frac{1}{3} \sin 3x} \quad 10 \text{ pts}$

E) $y = \frac{x}{2} \sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$y' = \frac{1}{2} \sqrt{2-x^2} + \frac{x(-x)}{2 \cdot 2 \sqrt{2-x^2}} +$

$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{1-\frac{x^2}{2}}}$

$y' = \frac{1}{2} \sqrt{2-x^2} - \frac{x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2} \sqrt{2-x^2}}$

$= \frac{\sqrt{2-x^2}}{2} - \frac{x^2}{2\sqrt{2-x^2}} + \frac{1}{\sqrt{2} \sqrt{2-x^2}}$

$= \frac{2-x^2-x^2+2}{2\sqrt{2-x^2}}$

$= \frac{2(2-x^2)}{2\sqrt{2-x^2}} = \sqrt{2-x^2} \quad 10 \text{ pts}$

III $y = \frac{3x}{8} - \frac{3\sin 10x}{80} - \frac{\sin^5 5x \cos 5x}{20}$

$$y' = \frac{3}{8} - \frac{3\cos 10x}{8} - \frac{1}{20} [3 \cdot 5 \cos^2 5x \sin^2 5x - 5 \sin^4 5x]$$

$$\frac{3}{8} - \frac{3(\cos^2 5x - \sin^2 5x)}{8} - \frac{1}{4} (3 \cos^2 5x \sin^2 5x) + \frac{1}{4} \sin^4 5x$$

$$\frac{3}{8} - \frac{3}{8} (1 - 2 \sin^2 5x) - \frac{1}{4} (3 \sin^2 5x (1 - \sin^2 5x) + \frac{1}{4} \sin^4 5x$$

$$\frac{3}{8} - \frac{3}{8} + \frac{3}{4} \sin^2 5x - \frac{3}{4} \sin^2 5x + \frac{3}{4} \sin^2 5x + \frac{1}{4} \sin^4 5x$$

$$= \sin^4 5x \text{ 10 PTS}$$

IV WTP $\sec^{-1} x = \cos^{-1}(\frac{1}{x})$.

Let $y = \sec^{-1} x \quad |x| \geq 1 \quad 0 \leq y \leq \pi, y \neq \frac{\pi}{2}$

$\sec y = x$

$\frac{1}{\cos y} = x \quad 10 \text{ pts}$

$\cos y = \frac{1}{x}$

$y = \cos^{-1}(\frac{1}{x})$

$\sec^{-1} x = \cos^{-1}(\frac{1}{x}) \text{ QED}$

Extra credit.

the derivative of f^{-1} at $x=2$

is $\frac{1}{f'(1)} = \frac{1}{\frac{1}{3}} = 3$.

"at $x=2$ " i.e. $f(\text{what})=2$

Answer: 1.

Formula is

$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$

$(f^{-1})'(f(1)) = \frac{1}{f'(1)}$

$(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{3}} = 3$

V

A) Since f passes 1st the vertical line test then passes the horizontal line test we know f an inverse f^{-1} . 5pts

B) Since $D_f = R_{f^{-1}}$ and $R_f = D_{f^{-1}}$ and $R_f \in [0, 4]$ then $D_{f^{-1}} \in [0, 4]$ 5pts

C) $f^{-1}(f^{-1}(3))$

$f^{-1}(3)$ is asking $f(\text{what})$ yields 3
Answer is 4.

Now problem becomes

$f^{-1}(4)$, again $f(\text{what})$ yields 4

Answer: 5.

5PTS

$\therefore f^{-1}(f^{-1}(3)) = 5$

1988

100	2	104 }
90	4	
86	9	
70	5	
60	④	
59	4	36
78.54		

The bottom is
sinking even lower.

3 of the 4 bottoms
do not have the
No real effort
were so-so last year -
they have met their
limit.

others doing very well
their average 85.7

We do put almost all of
Review exams on
the Boards. I plan to
continue this - works
great.

85

100	2
90	6
80	8
70	10
60	1
54	1
83.11	

102
57

10/25/85

Excellent exam.

Last section was ideal
after "The Home Quiz"
No changes.

10/25/91

100	0	} 28
90	7	
80	2	
70	6	
60	1	
59	0	66

1334 83,4
16

We did lots of board work -
I had students put up almost
all the review problems as
well as "last year's exam"
Almost ~~not~~ ^{no} slack time in class
at all.

Advanced Placement Calculus

Copy original problem.

Convince me that you understand the concept!

no decimals - f course!

Chapter 5 Exam

Name

Calc ch5

Per

Date

10/31/96
10/28/99

I

Given $f(x)$, find $f'(x)$. Simplify answers.

(10 pts ea)

a) $f(x) = -\frac{1}{3} \sin^{-1}\left(\frac{3}{x}\right)$

b) $f(x) = \ln[\ln(1+x^2)]$

c) $f(x) = (2^{3x})(5^{7x})$

d) $f(x) = 3^{\frac{1}{3} \sin 3x}$

e) $f(x) = \frac{x}{2} \sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

II

Given $y = 3^x$. Find the x -intercept of the tangent to the curve y at $x = 2$.

(15 pts)

III

Prove that $y' = \sin^4 5x$, given : $y = \frac{3x}{8} - \frac{3 \sin 10x}{80} - \frac{\sin^3 5x \cos 5x}{20}$

(10 pts)

IV

Show that $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$. Be sure to include all restrictions on any variables.

(10 pts)

V

Given the information shown on the graph of $f(x)$.

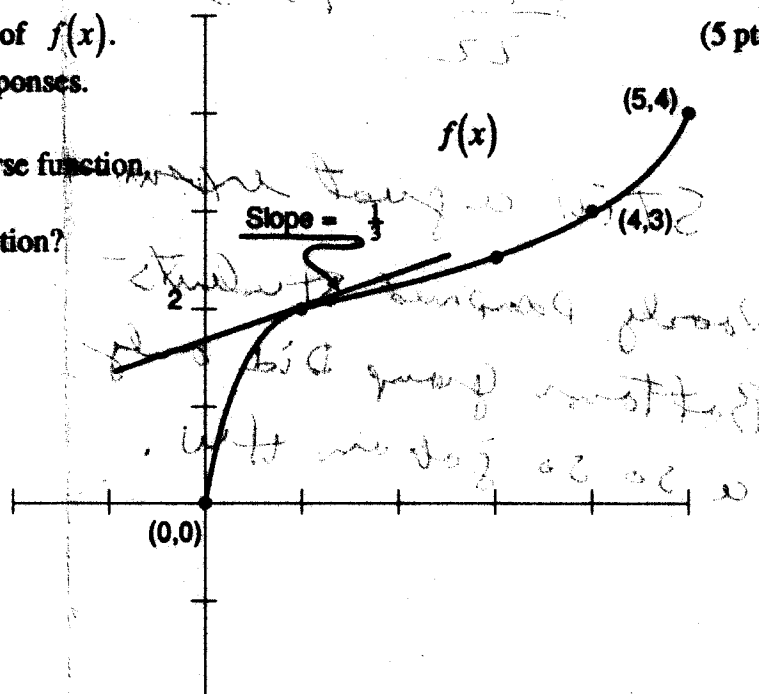
Be sure you explain and/or justify your responses.

(5 pts ea)

a) How do you know that $f(x)$ has an inverse function?

b) What is the domain of this inverse function?

c) Find $f^{-1}(f^{-1}(3))$.



Extra Credit ----- 5 pts -----

Find the derivative of $f^{-1}(x)$ as defined in section V evaluated at $x = 2$. Explain.

10/28/99

96

100	2
90	5
80	6
70	4
60	1
59	5

101



22

100	1
90	3
80	8
70	5
60	2
59	4

103



55

$$\frac{1789}{23} = 77.8$$

$$\frac{1722}{22} = 78.3$$

Looks like the Pretenders are sinking to the bottom. actually they are able but no self discipline

Still a great exam - Poorly prepared students Bottom group did only a so so job on the.

"Continuous Enjoyment -- The Third Time is the Charm"

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

- 1) Given the function $G(x)$ as graphed on the right, which of the following statements are true?

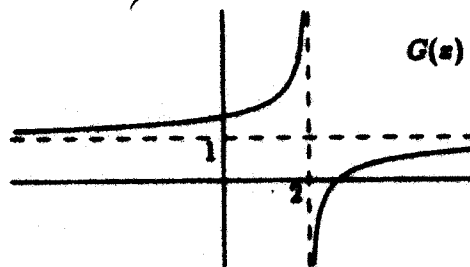
I. $x = 2$ is a vertical asymptote of $G(x)$

because $\lim_{x \rightarrow 2^-} G(x) = +\infty$.

II. $x = 2$ is a vertical asymptote of $G(x)$

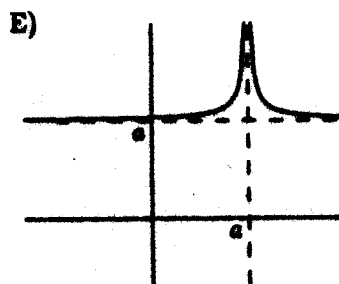
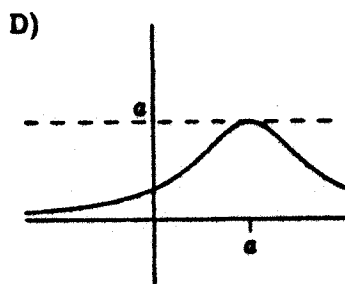
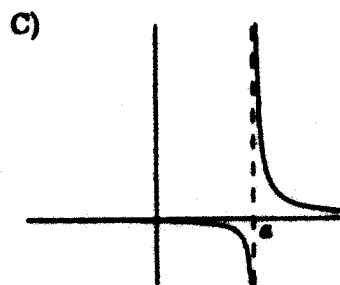
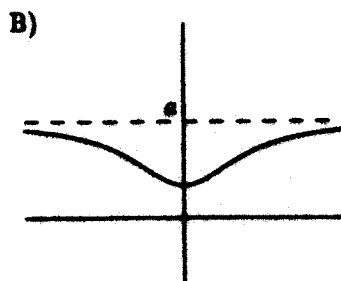
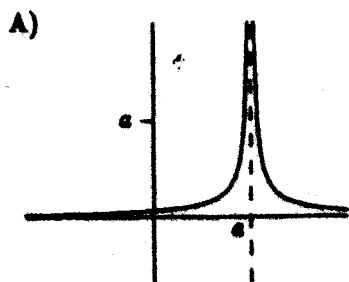
because $\lim_{x \rightarrow 2^-} G(x) = \lim_{x \rightarrow 2^+} G(x)$

III. $\lim_{x \rightarrow 2^+} G(x) = -\infty$

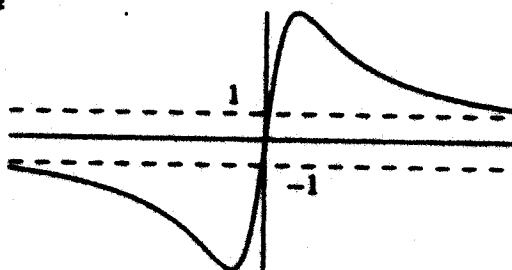


- a) I only b) II only c) III only d) I and III only e) I, II, and III

- 2) Which is a possible graph of $f(x) = \frac{a}{(a-x)^2}$?

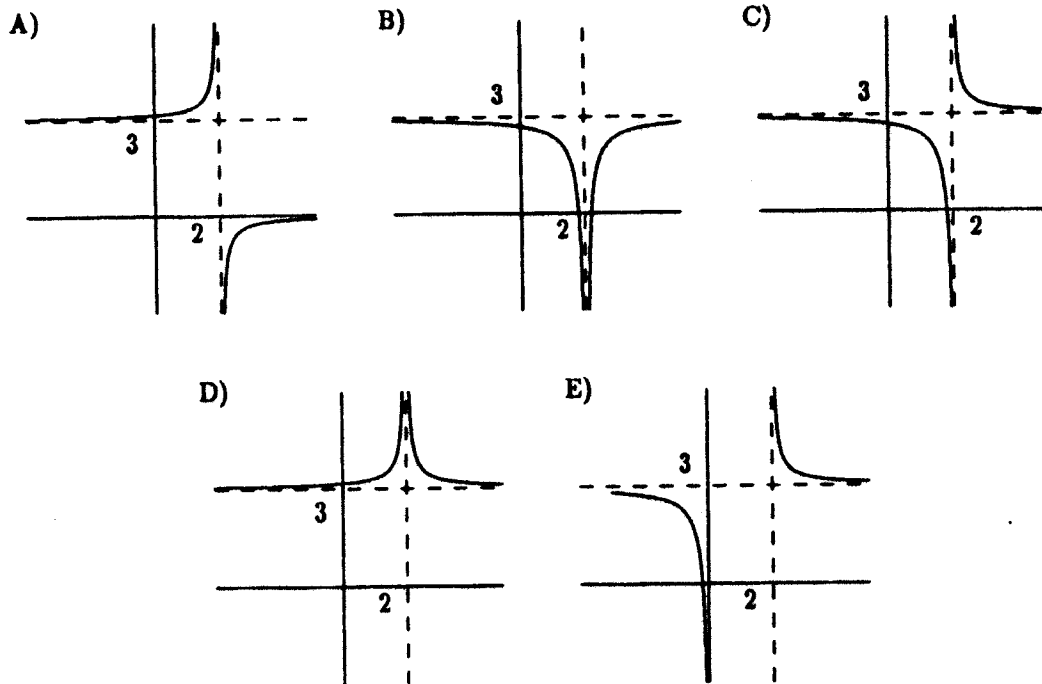


- 3) On the graph of $f(x)$ on the right, $y = 1$ and $y = -1$ are horizontal asymptotes. Therefore, $\lim_{x \rightarrow -\infty} f(x) =$



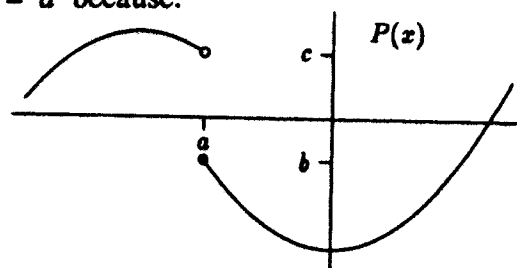
- a) -1 b) 0 c) 1 d) 2 e) Limit DNE

- 4) A possible graph for a function $f(x)$ such that $\lim_{x \rightarrow 2^-} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 3$ is:



- 5) The function $P(a)$ graphed on the right is not continuous at $x = a$ because:

- I. $\lim_{x \rightarrow a} P(a)$ does not exist
- II. $P(a)$ is not defined
- III. $P(a) \neq c$



- a) I only b) II only c) III only d) I and III only e) I and II only

- 6) Which of the following functions are continuous for all real numbers?

- I. $y = 2x^{4/3}$
- II. $y = \sqrt[3]{3x^2 - 1}$
- III. $y = \frac{2x + 5}{2x^2 + 5}$

- a) None b) I only c) II only d) I and II only e) All

- 7) Given $f(x) = \begin{cases} 3x - 1 & \text{if } x < -3 \\ -x^2 - 1 & \text{if } -3 \leq x \leq 2 \\ 3x + 1 & \text{if } x > 2 \end{cases}$ which of the following statements are true?

- I. $f(x)$ is continuous at $x = -3$
- II. $f(-3) = \lim_{x \rightarrow -3} f(x)$
- III. $f(2) = \lim_{x \rightarrow 2} f(x)$

- a) I only b) II only c) III only d) I and II only e) II and III only

- 8) $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$ then $(f \circ g)(x)$ is continuous for

- a) $(-\infty, \infty)$ b) $(0, \infty)$ c) $[0, \infty)$ d) $(-\infty, -2) \cup (2, \infty)$ e) $(2, \infty)$

- 9) Select the function below which is continuous for all real numbers except $x = 1$ and $x = -1$.

a) $f(x) = \frac{x^2 + 4}{(x-1)(x+2)}$ b) $f(x) = \frac{3x+1}{\sqrt{x}}$ c) $f(x) = \frac{2x-1}{x^2+1}$

d) $f(x) = \frac{2x}{x^2-1}$ e) $f(x) = \frac{x^2-1}{x^2+1}$

- 10) If $f(x) = \begin{cases} 2x-3 & \text{for } x > -1 \\ kx^3 & \text{for } x \leq -1 \end{cases}$ then for $f(x)$ to be continuous everywhere, k must equal:

- a) 0 b) 1 c) -1 d) 5 e) -5

- 11) Which of the following statements is false if $f(x) = \frac{1}{x}$, $g(x) = x^2$, and $a \neq 0$?

a) $\lim_{x \rightarrow a} f(g(x)) = \frac{1}{a^2}$ b) $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = a$ c) $\lim_{x \rightarrow a} f(f(x)) = \frac{1}{a}$

d) $\lim_{x \rightarrow a} g(g(x)) = a^4$ e) $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} = a^3$

- 12) Find the point or points on the graph of $y = x^3 + 3x$ where the tangent to the graph is parallel to $6x - y = 5$.

- a) (1, 0) or (-1, 0) b) (1, 4) c) (3, 0) d) (-1, -4) or (1, 4) e) (1, 1)

- 13) $D_x(\sqrt{x+1})$ is represented by which of the following:

a) $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{\Delta x + 1} - 1}{\Delta x}$ b) $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x+1}}{\Delta x}$ c) $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{x+1}}{\Delta x}$

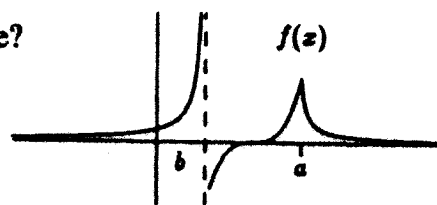
d) $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{\Delta x + 1}}{\Delta x}$ e) $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{x + \Delta x}}{\Delta x}$

- 14) Find the equation of the line perpendicular to $y = \sqrt{x+2}$ when $x = 2$.

- a) $y = -4x + 8$ b) $x - 4y = -6$ c) $4x + y = 6$ d) $x + 4y = 10$ e) $4x + y = 10$

- 15) Given the function graphed on the right, which of the following is true?

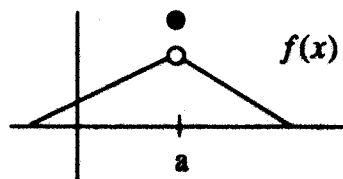
- I. $f(x)$ is not continuous at $x = b$.
 II. $f'(a^+) \neq f'(a^-)$.
 III. $f(x)$ is not differentiable at $x = b$.



- a) I only b) II only c) III only d) II and III only e) I, II, and III

- 16) Which of the following statements is true about the graph of $f(x)$ shown on the right?

- I. $f(x)$ exists at $x = a$.
 II. $f(x)$ is continuous at $x = a$.
 III. $f(x)$ is differentiable at $x = a$.



- a) I only b) II only c) III only d) I and II only e) I, II, and III

- 17) If $f(x) = (x^2 + 4)(3x^3 - 2)$, then $\frac{df}{dx} =$
- a) $18x^3$ b) $15x^4 + 36x^2 - 4x$ c) $12x^4 + 36x^2 + 4x$
d) $18x^4 + 36x^2 - 4x$ e) $18x^3 + 36x^2 - 4x$
- 18) If $f(x) = x^{\frac{3}{5}} - x^{\frac{2}{5}}$ for what values of x will $f(x)$ have horizontal tangents?
- a) 3 b) 2 c) $\frac{2}{3}$ d) $\frac{3}{5}$ e) $\frac{2}{5}$
- 19) Find an equation of a line tangent to $y = x^4 - 8x$ that is parallel to $y + 4x = 2$.
- a) $y = -7$ b) $x = 1$ c) $4x + y = 29$ d) $x + 4y = 29$ e) $4x + y = -3$
- 20) Given $f(x) = \frac{2}{x} - \frac{3}{x^2} + \frac{5}{3x^3}$. Find $f'(x)$.
- a) $2 + \frac{6}{x^3} - \frac{5}{x^4}$ b) $-\frac{2}{x^2} + \frac{6}{x^3} - \frac{5}{x^4}$ c) $-\frac{2}{x^2} - \frac{6}{x^3} - \frac{5}{x^4}$ d) $-\frac{2}{x^2} + \frac{6}{x^3} + \frac{5}{x^4}$ e) $-\frac{2}{x^2} - \frac{6}{x^3} + \frac{5}{x^4}$
- 21) Given $y = \frac{1-3x}{2x-5}$. Find $\frac{dy}{dx}$.
- a) $-\frac{3}{2}$ b) $\frac{-12x+17}{(2x-5)^2}$ c) $\frac{13}{(2x-5)^2}$ d) $\frac{-17}{(2x-5)^2}$ e) $\frac{-3}{(2x-5)^2}$
- 22) Find $D_x \left[\frac{x^2 - 5x + 6}{x - 3} \right]$
- a) $2x - 5$ b) $\frac{3x^2 - 16x + 21}{(x-3)^2}$ c) $-\frac{3x^2 - 16x + 21}{(x-3)^2}$ d) $5x - 2$ e) 1
- 23) If $f(x) = \frac{x^2 + 2x}{x - 1}$, then $f'(0) =$
- a) 2 b) 6 c) 14 d) -2 e) 3
- 24) Find the derivative of the function $f(x) = \frac{x^2}{3} - \frac{3}{x^2}$.
- a) $\frac{4x^2 - 9}{6x}$ b) $\frac{2x^4 + 18}{3x^3}$ c) $\frac{x^3 - 9}{3x}$ d) $\frac{2x^4 - 18}{3x^3}$ e) $\frac{18 - 2x^4}{3x^3}$
- 25) If $f(x) = \sqrt{x - \sqrt{x}}$, then $f'(x) =$
- a) $\frac{1}{2\sqrt{x}\sqrt{x - \sqrt{x}}}$ b) $\frac{\sqrt{x}}{2\sqrt{x}\sqrt{x - \sqrt{x}}}$ c) $\frac{2\sqrt{x} - 1}{4\sqrt{x}\sqrt{x - \sqrt{x}}}$ d) $\frac{1}{\sqrt{x - \sqrt{x}}}$ e) $\frac{\sqrt{x - \sqrt{x}}}{2(2\sqrt{x} - 1)}$

- 26) The equation of the horizontal tangent line to $f(x) = \frac{x^2}{2} + x - 1$ is
- a) $y = -1$ b) $y = 0$ c) $y = -\frac{3}{2}$ d) $y = -\frac{1}{2}$ e) $y = -\frac{3}{4}$
- 27) A particle is moving along a straight line according to the equation $s = -3 \cos 2t$ where $0 \leq t \leq 2\pi$. Find the intervals of time when the particle is moving to the left.
- a) $0 < t < \frac{\pi}{2}$ b) $\frac{\pi}{2} < t < \frac{3\pi}{2}$ c) $\pi < t < 2\pi$ d) $0 < t < \pi$ e) $\frac{\pi}{2} < t < \pi$ or $\frac{3\pi}{2} < t < 2\pi$
- 28) If $f(x) = 6 \sin 2x \cos 2x$ then $f'(\frac{\pi}{2}) =$
- a) 0 b) 3 c) 6 d) 12 e) -3
- 29) The slope of the normal line to the curve $f(x) = 2 \cos 4x$ at $\frac{\pi}{12}$ is:
- a) $-4\sqrt{3}$ b) $\frac{\sqrt{3}}{12}$ c) -4 d) $\frac{1}{4}$ e) $4\sqrt{3}$
- 30) If $f(x) = \frac{1 + \sin x}{\sin x - 1}$, then $f'(x) =$
- a) $\frac{2 \sin x \cos x}{(\sin x - 1)^2}$ b) $\frac{2 \cos x}{(\sin x - 1)^2}$ c) $\frac{2}{\cos x}$ d) $\frac{-2 \cos x}{(\sin x - 1)^2}$ e) 1
- 31) If $f(x) = \tan x + \sec x$, then $f'(\frac{\pi}{6}) =$
- a) 2 b) $4 + 2\sqrt{3}$ c) $\sqrt{3}$ d) $2 + \sqrt{3}$ e) $\frac{\sqrt{3}}{3}$
- 32) If $s(t) = \sin t \cos t$ describes the position of a particle moving on a straight line for time $t > 0$, what is the first time when the velocity of the particle is zero?
- a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{\pi}{2}$
- 33) The derivative of $f(x) = (x^2 - 4x + 4)^{35}$ is:
- a) $2x - 4$ b) $(x^2 - 4x + 4)^{34}$ c) $(2x - 4)(x^2 - 4x + 4)^{34}$ d) $70(x - 2)^{69}$ e) none of these
- 34) The equation of the line tangent to $f(x) = (3x^2 - 2x + 1)$ at $x = 0$ is:
- a) $y = -2x + 1$ b) $y = 1$ c) $x = 0$ d) $y = -\frac{1}{2}x + 1$ e) $y = \frac{1}{2}x + 1$
- 35) A particle moves in a straight line according to the equation $s(t) = (3t^2 - 2t)^5$. Find the instantaneous velocity of the particle at time $t = 1$.
- a) 1 b) 4 c) 20 d) 5 e) 0
- 36) The derivative of $\tan^3 2x$ is:
- a) $(\sec^2 2x)^3$ b) $3 \tan^2 2x$ c) $3 \tan^2 2x \sec^2 2x$ d) $6 \tan^2 2x \sec^2 2x$ e) $3(\sec 2x)^2$

37) Find the derivative of $(x^2 + 1)^{-3}$

- a) $-6x(x^2 + 1)^{-2}$ b) $-6x(x^2 + 1)^{-4}$ c) $-3(x^2 + 1)^{-4}$ d) $-3(x^2 + 1)^{-2}$ e) $-3(2x)^{-4}$

38) $\lim_{h \rightarrow 0} \frac{\tan^2(\frac{\pi}{4} + h) - \tan^2 \frac{\pi}{4}}{h} =$

- a) 2 b) $2\sqrt{2}$ c) 4 d) 8 e) does not exist

39) Let $y = \sqrt[3]{(x^2 - 6x + 9)^2}$, then $\frac{dy}{dx} =$

- a) $(x - 3)^{4/3}$ b) $\frac{4}{3}(x - 3)^{4/3}$ c) $\frac{2}{3(x^2 - 6x + 9)^{1/3}}$ d) $\frac{4(x - 3)}{3}$ e) $\frac{4(x - 3)(x^2 - 6x + 9)^{1/3}}{3}$

40) Which of the following functions are not differentiable at $x = 0$?

I. $f(x) = 3x^{2/3}$

II. $f(x) = 5x^{5/3}$

III. $f(x) = (x^2 - 1)^{3/2}$

- a) I only b) II only c) III only d) I and III only e) I, II, and III

41) $\frac{d}{dx} \left(2\sqrt{x} - \frac{3}{\sqrt{x}} + \frac{3}{\sqrt[3]{x}} \right)$ evaluated at $x = 1$ is:

- a) 2 b) $-\frac{3}{2}$ c) $\frac{3}{2}$ d) -2 e) 4

42) $f(x) = \sin \sqrt{2x}$, then $f'(x) =$

- a) $\cos \sqrt{2x}$ b) $\frac{\cos \sqrt{2x}}{\sqrt{2x}}$ c) $\frac{2 \cos \sqrt{2x}}{\sqrt{2x}}$ d) $\frac{\sin \sqrt{2x}}{\sqrt{2x}}$ e) $\frac{-\cos \sqrt{2x}}{\sqrt{2x}}$

43) $D_t \sqrt{\frac{t^2 + 1}{2t - 3}} =$

- a) $\sqrt{\frac{2t - 3}{t^2 + 1}} \left(\frac{t^2 - 3t - 1}{(2t - 3)^2} \right)$ b) $\sqrt{\frac{2t - 3}{t^2 + 1}} \left(\frac{3t^2 - 3t + 2}{(2t - 3)^2} \right)$ c) $\sqrt{\frac{t^2 + 1}{2t - 3}} \left(\frac{t^2 - 3t - 1}{(2t - 3)^2} \right)$
d) $\frac{1}{2} \sqrt{\frac{2t - 3}{t^2 + 1}}$ e) $\sqrt{\frac{2t + 1}{2}}$

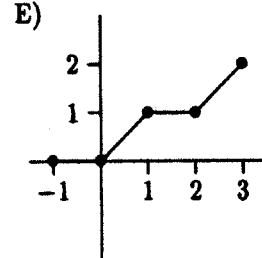
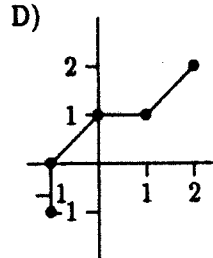
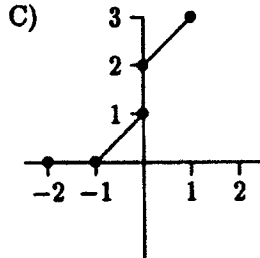
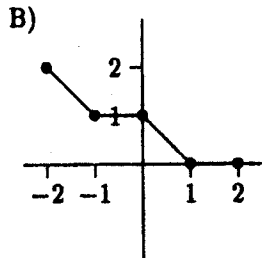
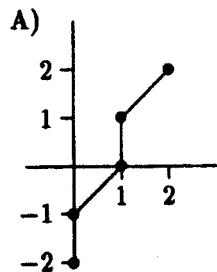
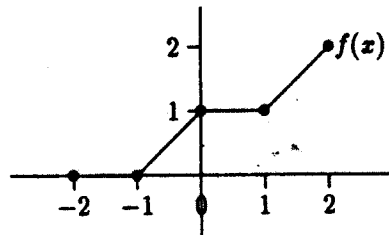
44) If $f(1) = 4$ and $f'(1) = 3$ and $g(x) = \sqrt{f(x)}$, then $g'(1) =$

- a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{3}{2}$ e) 2

45) If $y = \cos^2 x - \sin^2 x$ then $\frac{d^2 y}{dx^2} =$

- a) 0 b) $4 \cos x \sin x$ c) $-2 \sin 2x$ d) $4 \sin 2x$ e) $-4 \cos 2x$

46. If the graph of $f(x)$ is as shown on the right, then the graph of the inverse relation of $f(x)$ is:



47. The value of k for which $f(x) = \frac{x-2}{x+k}$ is its own inverse is:

a) 2 b) -2 c) -1 d) 1 e) 0

48. Which of the following functions are their own inverses?

I. $f(x) = \frac{1}{x}$

II. $f(x) = \frac{1}{x^3}$

III. $f(x) = x$

a) I only b) II only c) III only d) I and III only e) II and III only

49. If $f(x) = \frac{x+1}{x-2}$ then $f^{-1}(x) =$

a) $\frac{x-2}{x+1}$ b) $\frac{2x+1}{x-1}$ c) $\frac{2x-1}{x+1}$ d) $\frac{x-1}{2x+1}$ e) There is no inverse

50. Which of the following functions are one-to-one?

I. $f(x) = x^3 - x^2$

II. $f(x) = x^3 - 2x$

III. $f(x) = x^3 + 2$

a) I only b) II only c) III only d) I and III only e) II and III only

51. If $f(x) = \frac{1}{x}$ and $f(g(x)) = x$ then $g(x) =$

a) x b) x^2 c) $\frac{1}{x}$ d) $\frac{1}{x^2}$ e) $-x$

52. If $f(x_0) = 2$, $f'(x_0) = 3$, and $f(x)$ is an increasing function, find the derivative of f^{-1} evaluated at $x_0 = 2$.

a) $\frac{1}{2}$ b) 2 c) 3 d) $-\frac{1}{3}$ e) $\frac{1}{3}$

53. If $f(x) = \sqrt[3]{x}$, then find the function which is the derivative of the inverse of f .

a) $\frac{\sqrt[3]{x}}{3x}$ b) $3x^2$ c) $2x$ d) $\sqrt{3x}$ e) x^3

54. $\log_2 e =$

a) 0 b) $\frac{1}{2}$ c) 1 d) 2 e) 4

55. If $f(x)$ is a one-to-one continuous function with the values given in the table on the right, then for the inverse function,

x	$f(x)$	$f'(x)$
1	2	4
2	5	6

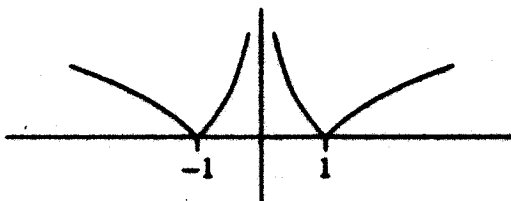
f^{-1} , the derivative of f^{-1} at $x = 2$ is :

- a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{5}{6}$ e) $-\frac{1}{25}$
56. For what values of x does $\ln x^4 = 4 \ln x$?
- a) all real numbers b) $x \geq 0$ c) $x > 0$ d) $x \geq 1$ e) $x \geq e$

57. If $f(x) = \sqrt{\ln x^3}$, then $f'(x) =$

- a) $\frac{3}{2x}$ b) $\frac{1}{x\sqrt{x}}$ c) $\frac{\sqrt{3}}{2x\sqrt{\ln x}}$ d) $\frac{\sqrt{3}}{2\sqrt{\ln x}}$ e) $\sqrt{\frac{3}{\ln x}}$

58. The graph shown on the right is most like:



- a) $f(x) = |\ln x|$ b) $f(x) = \frac{1}{|\ln x|}$ c) $f(x) = \ln|x|$ d) $f(x) = \frac{1}{\ln|x|}$ e) $f(x) = |\ln|x||$
59. If $f(x) = \ln(\cos 2x)$, then $f'(x) =$
- a) $\tan 2x$ b) $\cot 2x$ c) $-2 \tan 2x$ d) $-2 \cot 2x$ e) $2 \cot 2x$

60. $\frac{d}{dx} \ln(3-2x) =$

- a) $\frac{1}{3-2x}$ b) $\frac{2}{3-2x}$ c) $\frac{-2}{3-2x}$ d) $\frac{-1}{3-2x}$ e) $\frac{3}{3-2x}$

61. The equation of the line tangent to $y = \ln x$ at $x = e^2$ is:

- a) $y = \frac{1}{e^2}x + 1$ b) $y = \frac{1}{e^2}x$ c) $y = e^2x - 2$ d) $y = e^2x + 2$ e) $y = \frac{1}{e^2}x - 3$

62. If $f(x) = x \ln x$, then $f'(x) = 0$ when $x =$

- a) 1 b) 0 c) e d) e^2 e) $\frac{1}{e}$

63. $\lim_{x \rightarrow 0} \frac{\ln(2+x) - \ln 2}{x} =$

- a) $\ln \frac{1}{2}$ b) 0 c) $\frac{1}{2}$ d) $\ln 2$ e) e^2

64. $\frac{d}{dx} [\sin e^{2x}] =$

- a) $e^{2x} \cos e^{2x}$ b) $2e^{2x} \cos e^{2x}$ c) $\cos e^{2x}$ d) $-2x \cos e^{2x}$ e) $-e^{2x} \cos e^{2x}$

65. $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan^{-1} x =$

- a) ∞ b) $-\infty$ c) 0 d) $\frac{\pi}{2}$ e) $-\frac{\pi}{2}$

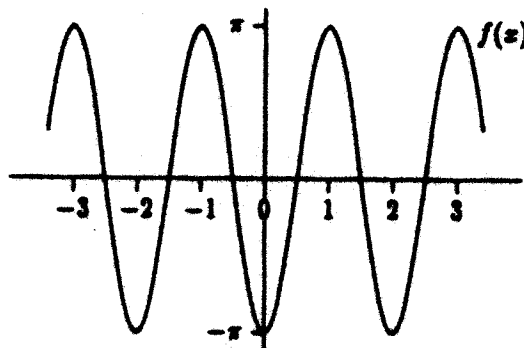
66. Find $\tan(\sin^{-1}(\frac{2}{3}) + \csc^{-1} 3)$

- a) $\frac{1}{2\sqrt{2}} - \frac{2}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{2}} + \frac{2}{\sqrt{3}}$ c) $\frac{4\sqrt{2} + \sqrt{3}}{2\sqrt{10} + 2}$ d) $\frac{4\sqrt{2} + \sqrt{3}}{2\sqrt{10} - 2}$ e) $\frac{3}{\sqrt{3} + 2\sqrt{3}}$

67. If $\tan \theta = 2$, find $\tan(\frac{\pi}{4} + \theta)$.

- a) $2 + \sqrt{3}$ b) $2 + \frac{\sqrt{3}}{3}$ c) $\frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$ d) $\frac{2 + \sqrt{3}}{1 + 2\sqrt{3}}$ e) $\frac{1 + \sqrt{3}}{2}$

68. The graph of $f(x)$ shown on the right is a variation of the $\cos x$ graph. Therefore, $f(x) =$



- a) $-\pi \cos x$ b) $\pi \cos(\pi x)$ c) $\pi \cos(\frac{x}{\pi})$ d) $-\pi \cos(\pi x)$ e) $-\pi \cos(\frac{x}{\pi})$

69. If $f(x) = \tan^{-1}(e^{2x})$ then $f'(x) =$

- a) $\frac{e^x}{\sec^2 e^x}$ b) $\frac{2e^{2x}}{1 + x^2}$ c) $\frac{2e^{2x}}{1 + e^{4x}}$ d) $\frac{4e^{4x}}{1 + e^{2x}}$ e) $\frac{e^{4x}}{1 + e^{4x}}$

70. $\sin(2 \cos^{-1} \frac{1}{4}) =$

- a) $\frac{5}{8}$ b) $\frac{\sqrt{15}}{4}$ c) $\frac{\sqrt{15}}{8}$ d) $\frac{\sqrt{15}}{16}$ e) $\frac{7}{8}$

71. If $f(x) = \tan^{-1}(\frac{1}{x})$ then $f'(\sqrt{3}) =$

- a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{\sqrt{3}}{4}$ e) $-\frac{\sqrt{3}}{6}$

72. The slope of the normal line to $f(x) = \tan^{-1} x$ at $x = 1$ is:

- a) $\frac{\pi}{4}$ b) $-\frac{4}{\pi}$ c) $\frac{1}{2}$ d) -2 e) $-\frac{1}{\sqrt{2}}$

73. If $f(x) = \cos^{-1} 5x$ then $f'(x) =$

- a) $\frac{-1}{\sqrt{1 - 25x^2}}$ b) $\frac{1}{\sqrt{1 - 25x^2}}$ c) $\frac{5}{\sqrt{1 - 25x^2}}$ d) $\frac{-5}{\sqrt{1 - 25x^2}}$ e) $-5 \sin^{-1} 5x$

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Convince *me* that you understand the concept!

Chapter 5 Exam

I Given $f(x)$, find $f'(x)$. Simplify, if possible. (10 pts ea)

a) $(\ln x)^{\ln x}$

b) $\sqrt{\ln x^3}$

II Given $f(1) = 3$, $f'(1) = 4$ and $g(x) = \sqrt{f(x)}$, find $g'(1)$. (10 pts)**III** Given $f(x) = \frac{x}{2}\sqrt{4x^2 + 3} + \frac{3}{4}\ln(2x + \sqrt{4x^2 + 3})$, show that $f'(x) = \sqrt{4x^2 + 3}$. (10 pts)**IV** Given $f(x) = x^5 + x - 22$ (Be very sure your method is clear.) (15 pts total)

a) Find $f(1)$

b) Find $f(2)$

c) Find $(f^{-1})'(12)$

V "Prove-it"s Be *especially* sure that your process is very clear to me! Be sure to include appropriate restrictions on *any* variables.

a) Prove: $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$. (15 pts)

b) Prove: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ (10 pts)

VI Given: $f(x) = \sin^{-1} x + \cos^{-1} x$ (20 pts tot)

a) Show that $f'(x) = 0$.

b) Explain the significance of your answer to part "a".

c) Let $\sin^{-1} x + \cos^{-1} x = k$. Solve for k .

Extra Credit -----5 pts-----

Solve the system of equations for the variables x and y :
$$\begin{cases} \ln xy = 3 \\ \ln \frac{x}{y} = 1 \end{cases}$$

$$I \quad \ln \frac{x e^x}{\sqrt{1+x}}$$

1)

$$\ln x + \ln e^x - \frac{1}{2} \ln(1+x)$$

$$\ln x + x - \frac{1}{2} \ln(1+x)$$

$$\frac{1}{x} + 1 - \frac{1}{2(1+x)}$$

$$\frac{2+2x+2x+2x^2-x}{2x(1+x)}$$

$$\frac{2x^2+3x+2}{2x(1+x)}$$

b)

$$x^{\ln x}$$

$$y = x^{\ln x}$$

$$\ln y = (\ln x)(\ln x) = (\ln x)^2$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2 \ln x}{x}$$

$$\frac{dy}{dx} = 2x^{\ln x} \frac{\ln x}{x}$$

$$= 2x^{\ln x - 1} \ln x$$

$$c) y = \log_e 3^{-2}$$

$$y' = 0 \quad y \text{ is a constant}$$

$$d) \ln(\ln x)^2$$

$$\frac{2}{\ln x} \cdot \frac{1}{x} = \frac{2}{x \ln x}$$

$$e) x^e$$

$$e x^{e-1}$$

$$II \quad f(x) = \ln 2x$$

$$y = \ln 2x$$

$$x = \ln 2y$$

$$e^y = 2x$$

$$\frac{1}{2} e^x = y$$

$$y' = \frac{1}{2} e^x \bigg|_{x=1} = \frac{1}{2} e$$

OR

$$f = \ln 2x \quad f' = \frac{2}{2x} = \frac{1}{x}$$

$$g = ? \quad g'(f(x)) = \frac{1}{f'(x)}$$

g is inverse set

$$III \quad f(x) = \ln(\ln x)$$

$$1) D_f \quad \ln x > 0 \text{ where } x > 1$$

$$\therefore D_f \quad x > 1 \quad x > 0 \text{ no domain } -5$$

$$b) f'(\frac{1}{e}) = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$= \frac{1}{\ln(\frac{1}{e})} \cdot \frac{1}{\frac{1}{e}} \quad (1)$$

$$= (-1) e = -e$$

$$f(x) = \ln 2x \quad \text{set } e = 2x \quad x = \frac{1}{2} e$$

$$g'(\frac{e}{2}) = \frac{1}{\frac{1}{\frac{1}{2} e}} = \frac{e}{2}$$

$$f'(e) =$$

$$\frac{1}{\ln x} \cdot \frac{1}{x} \quad x = e$$

$$(2) \frac{1}{1} \cdot \frac{1}{e} = e^{-1}$$

$$\text{IV } f(x) = x(\sin^{-1} 2x)^2 - 2x + \sqrt{1-4x^2} \sin^{-1} 2x$$

$$f'(x) = (\sin^{-1} 2x)^2 + 2x \frac{(\sin^{-1} 2x) \cdot 2}{\sqrt{1-4x^2}} - 2 + \frac{\sin^{-1} 2x \cdot (-4x)}{\sqrt{1-4x^2}} + \frac{\sqrt{1-4x^2} \cdot 2}{\sqrt{1-4x^2}}$$

$$f' = (\sin^{-1} 2x)^2$$

$$\text{V } 4) \sin \tan^{-1} x = \frac{x}{\sqrt{1+x^2}}$$

$$\text{let } \tan^{-1} x = u \quad |u| \leq \frac{\pi}{2}, x \in \mathbb{R}$$

$$\tan u = x$$

$$\sin^2 u = x^2 \cos^2 u$$

$$\sin^2 u = x^2 (1 - \sin^2 u)$$

$$\sin^2 u (1+x^2) = x^2$$

$$\sin^2 u = \frac{x^2}{1+x^2}$$

$$\sin(\tan^{-1} u) = \frac{\pm x}{\sqrt{1+x^2}}$$

$$\sin(\tan^{-1} u) = \frac{x}{\sqrt{1+x^2}}$$

when $u \in \text{Quadrant I}$, $x > 0$
and $\sin > 0$,

when $u \in \text{Quadrant II}$, $x < 0$
and $\sin < 0$! Do not
need extra neg sign.

$$\text{B) } f = \sec^{-1} x \quad f' = ?$$

$$g = \sec x \quad g' = \sec x \tan x$$

$$f'(x) = \frac{1}{g'(f(x))}$$

$$= \frac{1}{\sec(\sec^{-1} x) \tan(\sec^{-1} x)}$$

$$= \frac{1}{x \cdot \frac{x\sqrt{x^2-1}}{|x|}}$$

$$= \frac{1}{|x|\sqrt{x^2-1}}$$

$$\text{VI } f(x) = \tan^{-1} x + \cos^{-1} x$$

$$\tan^{-1} x + \cos^{-1} \frac{1}{x}$$

$$\text{a) } f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \left(-\frac{1}{x^2}\right) + \frac{1}{\frac{x^2+1}{x^2}} \cdot \left(-\frac{1}{x^2}\right)$$

$$\text{b) } f \text{ is a constant.}$$

$$\text{c) } \tan(\quad) = \tan k$$

$$\text{① } \frac{x + \frac{1}{x}}{1 - x \cdot \frac{1}{x}} \leftarrow \neq 0$$

$$\text{② } \cos(\quad) = \cos k$$

$$\cos \tan^{-1} x \cos \tan^{-1} \frac{1}{x} - \sin \tan^{-1} x \sin \tan^{-1} \frac{1}{x} =$$

$$\frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+\frac{1}{x^2}}} - \frac{x}{\sqrt{1+x^2}} \cdot \frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}} =$$

$$0 = \cos k$$

$$\therefore k = \frac{\pi}{2}$$

$$\frac{\sin \cos^{-1}(\frac{1}{x})}{\cos(\cos^{-1}(\frac{1}{x}))} = \frac{\sqrt{1-\frac{1}{x^2}}}{\frac{1}{x}} =$$

$$x \frac{\sqrt{x^2-1}}{|x|}$$

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Marie Suga 87
Deke Shelton 58
Becky Wylie 59
Roy Barker 79
Kim Gilmore 82
Chris Bush 98

11/7/96

Don Zuercher 89
Sasha Bejell 68
Erika Dangavetis 67
Jagjeet Khalsa 51

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Bridget Lewis 83
Renata Bradsky 75
Yohan Hong 72
Tara Branson 70
Jon Gustis 52

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GRIN Rishner 85
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Alan Savage 64
Tiffany Hall 61
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Chapter 5 Make-Up Exam

I Given $f(x)$, find $f'(x)$. Simplify, if possible.

(5 pts ea)

a) $\ln \frac{xe^x}{\sqrt{1+x}}$

b) $x^{\ln x}$

c) $\log_4 3^{-2}$

d) $\ln(\ln x)^2$

e) x^e

II Given $f(x) = \ln 2x$. Find the derivative of the inverse function of f at $x = 1$

(10 pts)

III Given $f(x) = \ln(\ln x)$

(10 pts tot)

a) Determine the domain of f .

b) Find

- 1) $f'\left(\frac{1}{e}\right)$.
- 2) $f'(e)$

IV Given $f(x) = x(\sin^{-1} 2x)^2 - 2x + \sqrt{1-4x^2} \sin^{-1} 2x$, show that $f'(x) = (\sin^{-1} 2x)^2$.

(10 pts)

V "Prove-it"s Be *especially* sure that your process is very clear to me! Be sure to include appropriate restrictions on *any* variables.

a) Prove: $\tan(\sec^{-1} u) = \frac{x\sqrt{x^2-1}}{|x|}$.

(10 pts)

b) Prove: $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

(10 pts)

VI Given: $f(x) = \tan^{-1} x + \cot^{-1} x = \tan^{-1} x + \tan^{-1} \frac{1}{x}$

(25 pts tot)

a) Show that $f'(x) = 0$.

b) Explain the significance of your answer to part "a".

c) Let $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = k$

1) Why can you *not* use the "tangent add" formula with this? (hint: Try it and find out.)

2) Solve for k .

I

A) $y = \ln(\cos x + \ln x)$ B) $y = \sin^{-1} e^x$ C) $y = x^2(2x^2 - x)^{\frac{1}{2}}$

$y' = \frac{-\sin x + \frac{1}{x}}{\cos x + \ln x}$ $y' = \frac{e^x}{\sqrt{1 - e^{2x}}}$ $y' = 2x(2x^2 - x)^{\frac{1}{2}} + \frac{x^2(4x - 1)}{2\sqrt{2x^2 - x}}$

$= \frac{1 - x \sin x}{x(\cos x + \ln x)}$ $= \frac{4x(2x^2 - x) + 4x^3 - x^2}{2\sqrt{2x^2 - x}}$

$= \frac{8x^3 - 4x^2 + 4x^3 - x^2}{2\sqrt{2x^2 - x}}$

$= \frac{12x^3 - 5x^2}{2\sqrt{2x^2 - x}}$

D) $y = \sqrt{x + \sqrt{x}}$ E) $y = \cos^{-1}\left(\frac{2}{x}\right)$

$y' = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}$ $y' = \sin^{-1}\left(\frac{x}{2}\right)$

$= \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$ $y' = \frac{1}{\frac{1}{2}\sqrt{\frac{x^2}{4} - 1}} \cdot \frac{1}{2}$

$= \frac{1}{1x\sqrt{\frac{x^2 - 4}{2}}}$

$= \frac{2}{1x\sqrt{x^2 - 4}}$

F) $y = \sqrt{x - \sin^2 x}$

$y' = \frac{1 - 2x \cos x^2}{2\sqrt{x - \sin^2 x}}$

G) $y = \left(\frac{1}{x}\right)^{\frac{1}{x}}$ I) $y = \ln x^{\ln x}$

$y = x^{-\frac{1}{x}}$ $y = (\ln x) \ln x$

$y = e^{\ln x \cdot -\frac{1}{x}}$ $= (\ln x)^2$

$= e^{-\frac{1}{x} \ln x}$ $y' = \frac{2 \ln x}{x}$

$y' = \left(\frac{1}{x}\right)^{\frac{1}{x}} \left[\frac{\ln x}{x^2} - \frac{1}{x^2} \right]$

$\frac{1}{x^2} \left(\frac{1}{x}\right)^{\frac{1}{x}} [\ln x - 1]$

H) $y = \log_9 3^x$

$= \log_9 9^{\frac{x}{2}}$

$= \frac{x}{2} \log_9 9$

$= \frac{x}{2}$

$y' = \frac{1}{2}$

III) $y = \frac{x}{2} - \frac{\sin 2x}{4}$

$y' = \frac{1}{2} - \frac{2}{4} \cos 2x$

$= \frac{1}{2} (1 - \cos^2 x + \sin^2 x)$

$= \frac{1}{2} (2 \sin^2 x)$

$= \sin^2 x$

II

$f(x) = x^3$ $g(x) = \sin x$ $h(x) = x^5$

$f'(x) = 3x^2$ $g'(x) = \cos x$ $h'(x) = 5x^4$

$k(x) = f(g(h(x)))$

$k'(x) = f'(g(h(x))) g'(h(x)) h'(x)$

$= 3(\sin x^5)^2 \cos x^5 \cdot 5x^4$

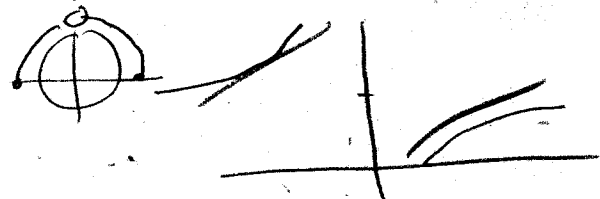
$= 15x^4 \sin^2 x^5 \cos x^5$

IV

$$y = \sec^{-1} x$$

$$|x| \geq 1$$

$$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$$



$$\sec y = x$$

$$\sec y \tan y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$= \frac{1}{x \tan y}$$

$$= \frac{1}{x(\pm \sqrt{x^2 - 1})}$$

$$= \frac{1}{|x|(\sqrt{x^2 - 1})}$$

$$s^2 + c^2 = 1$$

$$t^2 + 1 = \sec^2 y$$

$$\tan^2 y = \sec^2 y - 1$$

$$= x^2 - 1$$

$$\tan y = \pm \sqrt{x^2 - 1}$$

Notice tangent
is y' is > 0
in both areas.

Since y' must be > 0
if x and when $x > 0$
we use $+$, when
 $x < 0$ we must use
a minus.
we use $|x|$ instead
of just x

V

$$y = \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$$

$$y' = \tan^{-1} x + \frac{x}{1+x^2} - \frac{1}{2} \left[\frac{2x}{1+x^2} \right]$$

$$y' = \tan^{-1} x$$

VI

$$f(x) = \frac{(x+2)e^{x+2}}{x^2+5x+6}$$

$$= \frac{(x+2)e^{x+2}}{(x+2)(x+3)}$$

$$D_f x \neq \{-2, -3\}$$

∴ there is no tangent line at $x = -2$.

11/7/90

Lisa Hammond 88
 Jeff Hamer 86
 Amber Bain 85
 Gayle Moore 83
 Midori Allen 79
 Crystal McGee 76
 Kristel Laguna 67

1992

Susanneers 63

1994

Robin Holm 93
 Wesley Dickson 87
 Jane Cannon 86
 Amelia Ehardt 84
 Jeremiah Spadlin 80
 Dustin Baker 69

Great exam!

1997

Matt Dayavietis 89
 Oliver Hernandez 85

98

Liz Erwin 89
 Zee Hefari 89
 Nicole Scannell 86
 Nicole Gonzales 79
 Courtney Nicholas 72
 Brownyn Holman 69
 Andy Hahn 48

2000

Joe Mantell 89
 Lisa Erwin 89 *
 Robert Delgado 89 *
 Robert Green 84
 Ryan Hagerson 83 *
 Melvin Santanillo 82
 Matt Crouche 81 *
 * improved on previous exam.

only one
 who missed
 regular exam.

Advanced Placement Calculus AB

Copy the original problem.

Convince me that YOU understand the concept.

Name Key

Per 5

Date

CHAPTER #5 MAKE-UP

I Find Y' (ie. the first derivative) given $Y =$ (simplify final answers)

(5 pts ea)

A) $\ln(\cos x + \ln x)$

B) $\sin^{-1} e^x$

C) $x^2(2x^2 - x)^{\frac{1}{2}}$

D) $\sqrt{x + \sqrt{x}}$

E) $\cos^{-1}\left(\frac{2}{x}\right)$

F) $\sqrt{x - \sin x^2}$

G) $\left(\frac{1}{x}\right)^{\frac{1}{x}}$

H) $\log_3(x^x)$

I) $\ln x^{\ln x}$

II Given: $F(x) = x^3$, $G(x) = \sin x$, $H(x) = x^5$, and $K(x) = F(G(H(x)))$.

Find $K'(x)$

(10 pts)

III Given: $Y = \frac{x - \sin 2x}{2x^2 + 4x - 1}$. Show that $Y'(x) = \sin^2 x$.

(10 pts)

IV Given $y = \sec^{-1} x$. PROVE $y' = \frac{1}{|x| \sqrt{x^2 - 1}}$

(15 pts)

V Given $Y = x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2)$. Find $Y'(x)$ (simplified)

(10 pts)

VI Find the equation of the line tangent to $F(x)$ at $x = -2$

(10 pts)

when $F(x) = \frac{(x+2)e^{x+2}}{x^2 + 5x + 6}$

I

$$4) f(x) = (x-1)e^{2x} \quad 13) f(x) = \left[\frac{x+e^x}{e^x} \right]^3 \quad 3 \left[\frac{x+e^x}{e^x} \right]^2 \cdot \frac{(1-x)e^x}{e^{2x}}$$

$$e^{2x} + (x-1)2e^{2x} =$$

$$e^{2x}(1+2x-2) =$$

$$e^{2x}(2x-1)$$

$$u = \frac{x+e^x}{e^x}$$

$$\frac{du}{dx} = \frac{e^x(1+e^x) - (x+e^x)e^x}{e^{2x}}$$

$$\frac{(1-x)e^x}{e^{2x}}$$

$$\frac{3}{e^{3x}} [(x+e^x)^2 (1-x)]$$

$$c) f(x) = \ln(\ln x)^2$$

$$= 2 \ln(\ln x)$$

$$= \frac{2}{x \ln x}$$

$$d) f(x) = x^{\ln x}$$

$$x = e^{\ln x}$$

$$x^{\ln x} = e^{\ln x \ln x} = e^{(\ln x)^2} = e^u$$

$$\frac{du}{dx} = \frac{2 \ln x}{x} \therefore \left(\frac{2 \ln x}{x} \right) e^{(\ln x)^2} = \frac{2 \ln x}{x} x^{\ln x}$$

$$e) f(x) = \ln 3^x = x \ln 3$$

$$f' = \ln 3$$

$$f) \ln \frac{x e^x}{\sqrt{1+x}} =$$

$$g) f(x) = \ln \sqrt[3]{\cos x} = \frac{1}{3} \ln \cos x$$

$$f'(x) = \frac{-\sin x}{3 \cos x} = -\frac{1}{3} \tan x$$

$$\ln x + \ln e^x - \frac{1}{2} \ln(1+x) \quad H) f(x) = \log_{10} x^x = \frac{\ln x^x}{\ln 10} = \frac{x \ln x}{\ln 10}$$

$$\frac{1}{x} + 1 - \frac{1}{2(1+x)}$$

$$\frac{2+2x+2x+2x^2-1}{2x(1+x)}$$

$$\frac{2x^2+4x+1}{2x(1+x)}$$

$$I) f(x) = 10^{x^2} \quad 10 = e^{\ln 10} \quad 10^{x^2} = e^{x^2 \ln 10}$$

$$f'(x) = (2x \ln 10) 10^{x^2}$$

$$J) f(x) = x^x$$

$$x = e^{\ln x}$$

$$x^x = e^{x \ln x}$$

$$f'(x) = e^{x \ln x} [\ln x + 1] = x^x (\ln x + 1)$$

$$K) f(x) = 3 \sec^{-1} \frac{3}{x} \Rightarrow 3 \sec^{-1} u \quad du = -\frac{3}{x^2}$$

$$\frac{3}{x^2} \cdot \frac{-3}{x^2} = \frac{-9}{x^2 \left| \frac{3}{x} \right| \sqrt{\frac{9}{x^2} - 1}} =$$

$$\frac{-9}{x^2 \sqrt{9-x^2}} = \frac{-3}{\sqrt{9-x^2}} = f'(x)$$

$$I \quad f(x) = \cos(\sin^{-1} x) \quad u = \sin^{-1} x \quad f(u) = \cos u$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \frac{df}{dx} = \frac{-\sin(\sin^{-1} x)}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

$$II \quad y = (x^2 - 15)^6$$

$$y' = 6(x^2 - 15)^5 \cdot 2x = 48(16 - 15) = 48 = m$$

$$x = 4$$

$$48 \cdot 4$$

$$192$$

$$y - 1 = 48(x - 4)$$

$$y = 48x - 191$$

$$f(x) = \frac{1}{2} \left[x \sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right]$$

$$2f(x) =$$

$$2f'(x) = \sqrt{x^2 + a^2} + \frac{x^2}{\sqrt{x^2 + a^2}} + \frac{a^2(1 + \frac{x^2}{\sqrt{x^2 + a^2}})}{x + \sqrt{x^2 + a^2}}$$

$$= \frac{(\sqrt{x^2 + a^2})^2 + x^2}{\sqrt{x^2 + a^2}} + \frac{a^2(\sqrt{x^2 + a^2} + x)}{\sqrt{x^2 + a^2}(x + \sqrt{x^2 + a^2})}$$

$$2f'(x) = \frac{x^2 + a^2 + x^2 + a^2}{\sqrt{x^2 + a^2}} = 2\sqrt{x^2 + a^2}$$

$$f'(x) = \sqrt{x^2 + a^2}$$

$$IV \quad f(x) = \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x) \quad |x| \geq 1 \quad x \neq 0$$

$$\therefore f'(x) = \frac{1}{(x)\sqrt{x^2 - 1}}$$

its a good, tough makeup class

1981

87

86

78

76

73

71

61

60

40

91630

70.00

1982

82

76

72

70

41300

1983

93

88

75

70

1988 95

86

83

82

81

55

CALCULUS — CHAPTER 5 — MAKE-UP EXAM

NAME

Key 1981

1982

1983

1988

~~2000~~

1x151

I Given $F(x)$, find $F'(x)$.

(5 pts ea)

A) $F(x) = (x-1)e^{2x}$

B) $F(x) = \left[\frac{x+e^x}{e^x} \right]^3$

C) $F(x) = \ln(\ln x)^2$

D) $F(x) = x^{\ln x}$

E) $F(x) = \ln 3$

F) $\ln \frac{xe^x}{\sqrt{1+x}}$

G) $F(x) = \ln \sqrt[3]{\cos x}$

H) $F(x) = \log_{10} x^x$

I) $F(x) = 10^{x^2}$

J) $F(x) = x^x$

K) $F(x) = 3 \sec^{-1} \frac{3}{x}$

L) $F(x) = \cos(\sin^{-1} x)$

II Find the equation of the tangent line to the curve:

$$Y = (x^2 - 15)^6$$

at $(4, 1)$ (20 pts)

III Given $F(x)$, show that F' is as stated. (10 pts)

$$F(x) = \frac{1}{2} \left[x \sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right] ; \quad F'(x) = \sqrt{x^2 + a^2}$$

IV Given $F(x) = \cos^{-1} \left(\frac{1}{x} \right)$; Find $F'(x)$.

(10 pts)